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Development of stability control applied to a non-linear mechanism

Desarrollo de control de estabilidad aplicado a un mecanismo no lineal

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Abstract

This article presents the development of a stability control applied to a non-linear mechanism based on the principles of optimal control in order to offer a more appropriate alternative to the nature of non-linear systems. The developed control is based on the system called a rocker with engine and propeller which, despite not being a faithful model, represents one of the peculiarities of real non-linear systems such as unmanned aerial vehicles and that peculiarity is stability. For the realization of this control, the mathematical model of the rocker system with motor and propeller is considered as a starting point, which provides the mathematical equations and parameters necessary for the development of the control. Based on the information obtained from the mathematical model and the control that derives from it, simulations are developed that allow an analysis of the behavior of the control developed in the selected structure. In addition to this, we use our own computer tools to carry out this type of work, such as Matlab and Simulink, through which it will be possible to carry out the necessary models and simulations.

Control, Optimal, Stability

Resumen

Este artículo presenta el desarrollo de un control de estabilidad aplicado a un mecanismo no lineal basándose en los principios del control óptimo con el fin de ofrecer una alternativa más apropiada a la naturaleza de los sistemas no lineales. El control desarrollado se basa en el sistema denominado como balancín con motor y hélice el cual, a pesar de no ser un modelo fiel, representa una de las peculiaridades de sistemas no lineales reales como los son los vehículos aéreos no tripulados y dicha peculiaridad es la estabilidad. Para la realización de este control se considera como punto de partida el modelo matemático del sistema balancín con motor y hélice el cual proporciona las ecuaciones y parámetros matemáticos necesarios para el desarrollo del control. A partir de la información obtenida del modelo matemático y del control que deriva del mismo, se desarrollan simulaciones que permiten realizar un análisis del comportamiento del control desarrollado en la estructura seleccionada. Aunado a esto se utilizan herramientas computacionales propias para la realización de trabajos de esta índole como lo son Matlab y Simulink mediante las cuales será posible realizar los modelos y simulaciones necesarias.

Control, Óptimo, Estabilidad

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Introduction

With the current development of society, every day we are making more inroads in different sectors of the industry in order to provide new technologies that favor its development. The search for new technologies has led to the emergence of vehicles today known as UAVs (Unmanned Aerial Vehicles).

The UAVs are not a novelty since the first models date back to the year 1927 with the creation of the first drone which was the Standard E-1 [1], although the performance in terms of control was not the best, they served as point. Over time, advances have been made in UAVs, improving in different aspects, obtaining benefits that have promoted them as highly useful systems in different sectors, such as in environmental research, monitoring, deliveries in urban sectors [2] and even for complex applications such as 3D mapping [3], among others.

Due to the number of applications that these systems have, at present, the control and monitoring of the different activities with UAVs has become an area of great interest [4] and in the same way the characteristics that these vehicles have, such as: dimensions, range radius, flight stability control, etc. For the latter, special attention has been paid since it is important that these vehicles have the best possible stability. Therefore, it is important to pay attention to stability control, and this problem has been addressed since 1927 since the stability control of the first UAVs was not the best and was one of the first needs to be addressed since then.

Despite the advances that have been made in terms of control, PID control continues to be one of the most widely used in modern industry due to its practicality [5]. As a consequence of this, solutions to the stability problem have tended to use the PID control action, achieving good results, ranging from robust PID control projects [6] to PID control projects that act in conjunction with other control tools. as are the genetic algorithms [7] and the Kalman filter [8].

Although this gives some importance to PID control, the truth is that as these systems become more complex it is more difficult to determine the variables of the PID using other strategies so that this task becomes easier and, in addition, this control action is focused on linear systems, which does not make it ideal for applications such as the one stipulated in this article, which is why it is chosen to select an optimal control system [9] since it is like fuzzy control and mode control. Sliding [10] can be used in non-linear systems that are multiple inputs and multiple outputs [11] as is the case with UAVs and the system used as the basis for the development of the control shown in this article, which is a rocker arm with a motor and a propeller that, although it is not a faithful model to a UAV, represents one of its peculiarities which is stability.

Due to the above, the objective of this work is to develop a stability control for a non-linear mechanism, which in the case of this work is the system called as rocker with motor and propeller; based on the principles of an optimal control system; this control is intended to be an option that offers acceptable and reliable stability. In the same way, the development of the control system that is carried out in this project will be useful for those systems that are based on the same foundation, although these come to vary in terms of dimensions, weight, among other characteristics.

Methodology to be developed

The development of this research work includes the following steps:

- Obtain the mathematical model of the rocker system with motor and propeller.

- Development of the controller for the system.

- Simulations and analysis of the response obtained from the system controller, see Results.

- Analysis of the stability achieved with the controller, see Results.

I. Mathematical model of the rocker system with motor and propeller

The rocker system with motor and propeller is of a higher order, non-linear and unstable and by means of it the stability of a UAV can be simulated. Obtaining the mathematical model of the device is one of the first steps in the design of a control system [12]. For this task, an analysis of the parameters involved in the mechanical principle of the rocker is carried out, which consists of a support which works as a base for the axis through which the balancing action is exercised, in addition to this there is a DC motor and a propeller, which are the main elements of this system. The motor and its respective propeller are located at one end of the shaft and acting together are responsible for achieving the stability of the system. Said principle is shown in Figure 1 [5], where L represents the total length of the axis of rotation; θ the angular displacement; D_1 the length of the vertical bar; l the distance from the center of the axis of rotation to the motor; τ the torque; m_m the mass of the motor; g is the earth's gravitational acceleration constant; F_e the thrust force generated by the propellers; F_β the force produced by the coefficient of friction β and F_g the weight of the motor.

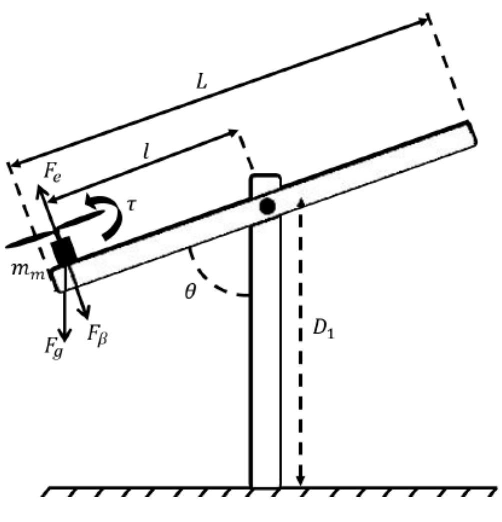

Figure 1 Principle of the rocker with motor and propeller
Source: Scientific Bulletin of the Institute of Basic Sciences and Engineering of the Autonomous University of the State of Hidalgo [5]

Once the variables of the system have been obtained, the equation that describes the movement that characterizes the system is obtained. For this action, Newton's law of rotational motion is used [13]; and from this law the following expression is obtained.

$$\sum \tau = I\,\alpha = I\,\ddot{\theta}(t) \quad (1)$$

From the Equation (1) it is determined that the total torque acting on a body τ is equal to the moment of inertia of the body I around the axis of rotation multiplied by its angular acceleration α [14].

The torque τ is defined as the tendency to produce a change in the rotational movement and is affected both by the magnitude of the force F and the lever arm r [15] also represented as l [16], and is defined as

$$\tau = F\,r = F\,l \quad (2)$$

In addition to the above, it is considered that the axis of inertia of the mobile bar is located right at its center of gravity and due to this the resulting force on the bar will be zero.

Due to the aforementioned, only the forces caused by the thrust of the propellers, the weight of the motor and also the friction produced in the axis of rotation are considered. Using the Equation (1) and taking into account the above, the behavior of the rocker system can be described with the following equation

$$\tau_m - \tau_{F_{gy}} - \tau_{F_\beta} = I_T\,\vec{\alpha} \quad (3)$$

Where τ_m is the torque caused by the movement of the propellers; $\tau_{F_{gy}}$ the torque caused by the weight of the motor; τ_{F_β} the torque caused by friction originated in the axis of rotation; I_T is the total moment of inertia of the system and $\vec{\alpha}$ the angular acceleration. Subsequently, the Equation (1) is substituted in the Equation (3), obtaining:

$$lF_e(t) - lF_g\theta(t) - lF_\beta\dot{\theta}(t) = I_T\,\ddot{\theta}(t) \quad (4)$$

Where F_e is the input given by the thrust force generated by the action of the rotation of the propellers. When the terms of the Equation (4) are ordered, the mathematical model of the system is obtained:

$$\ddot{\theta}(t) + \frac{lF_\beta}{I_T}\dot{\theta}(t) + \frac{lF_g}{I_T}\theta(t) = \frac{lF_e(t)}{I_T} \quad (5)$$

Once the mathematical model of the system has been obtained, the transfer function of the system is obtained:

ISSN 2523-6784

$$\frac{\ddot{\theta}(s)}{F_e(s)} = \frac{\frac{l}{I_T}}{s^2 + \frac{lF\beta}{I_T}s + \frac{lFg}{I_T}} \quad (6)$$

The Equation (6) corresponds to a second order system, which is described with the following expression:

$$\frac{Y(s)}{X(s)} = \frac{K}{s^2 + 2\delta\omega_n s + \omega_n^2} \quad (7)$$

Once the transfer function of Equation (7) is obtained, a conversion is carried out to obtain the model in state space. Obtaining the transfer function first is due to the fact that in the frequency domain the mathematical analysis is simpler. As a first step, all the terms of the transfer function are divided by the variable s of higher order obtaining in this way:

$$\frac{Y(s)}{X(s)} = \frac{Ks^{-2}}{1 + 2\delta\omega_n s^{-1} + \omega_n^2 s^{-2}} \quad (8)$$

Subsequently, the Equation (8) is separated in order to obtain two different expressions, one for the numerator and one for the denominator. In this way, once the separation is made, the following equations are obtained:

$$\frac{Y(s)}{E(s)} = Ks^{-2} \quad (9)$$

$$\frac{E(s)}{X(s)} = \frac{1}{1 + 2\delta\omega_n s^{-1} + \omega_n^2 s^{-2}} \quad (10)$$

From the resulting Equations (9) and (10) we solve for Y (s) and X (s) respectively, thus obtaining

$$Y(s) = Ks^{-2}E(s) \quad (11)$$

$$X(S) = E(s)(1 + 2\delta\omega_n s^{-1} + \omega_n^2 s^{-2}) \quad (12)$$

Once Equation (12) has been obtained, it is possible to obtain the state variables corresponding to this system, which are

$$s^{-2}E(s) = x_1 \quad (13)$$

$$\dot{x_1} = x_2 \quad (14)$$

$$\dot{x_1} = x - 2\delta\omega_n x_2 - \omega_n^2 x_1 \quad (15)$$

With the state variables the model in state space is obtained which is described with the following expressions

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\delta\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t) \quad (16)$$

$$y(t) = \begin{bmatrix}K & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (17)$$

For this work the following system data is considered l=0.3m; L=0.6; t_s=2.579seg; m_m=0.06kg; I_T=0.0108kg. m^2 obtained from the scientific bulletin of the Institute of Basic Sciences and Engineering of the Autonomous University of the State of Hidalgo [5]. With these data, the parameters ω_n, K and δ are determined with Equations (9) and (10).

As a first step, the value of ω_n is determined as follows:

$$\omega_n = \sqrt{\frac{lF_g}{I_T}} = \sqrt{\frac{(0.3m)(0.06kg)\left(9.81\frac{m}{s^2}\right)}{0.0108kgm^2}} = 4.04\frac{rad}{s} \quad (18)$$

The following expression is used to determine the value of the constant K, as indicated in the Equations (6) and (7).

$$K = \frac{l}{I_T} = \frac{0.3m}{0.0108kgm^2} = 27.77\left(\frac{1}{kgm}\right) \quad (19)$$

The following expression is used to calculate the parameter δ, obtaining:

$$\delta = \frac{4}{t_s\omega_n} = \frac{4}{(2.579seg)\left(4.04\frac{rad}{s}\right)} = 0.3839 \quad (20)$$

Finally, the values obtained in the Equations (16) and (17) are replaced to obtain the model in state space, remaining as follows.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (21)$$
$$y(t) = Cx(t) \quad (22)$$

Where A, B and C represent the following matrices.

$$A = \begin{bmatrix}0 & 1\\ -16.3216 & -3.101912\end{bmatrix} \quad (23)$$
$$B = \begin{bmatrix}0\\ 1\end{bmatrix} \quad (24)$$
$$C = \begin{bmatrix}27.77 & 0\end{bmatrix} \quad (25)$$

II. LQR controller design

The LQR control or quadratic linear regulator system is a multivariable controller that allows to obtain the best performance of the system.

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A control system is considered optimal when its different parameters are adjusted in such a way that the cost function (or also known as the performance index) reaches the lowest possible value [17]. Due to this, the different parameters of the optimal control are calculated based on this performance index. Therefore, the objective of the LQR controller is to minimize this performance index and bring it to the lowest possible values.

The minimization of the performance index is achieved through the application of feedback gains K. This feedback gain can be seen integrated into the performance index J in the following equation.

$$J = \int_0^\infty (x * Qx + u * Ru)dt \quad (26)$$

Where Q and R are real symmetric matrices and determine the relative importance of the error and the energy cost of the control signals, the Q matrix determines the importance of the weighted error and the R matrix the importance of the input [18].

The following equations are used to obtain the behavior index J.

$$\dot{x} = Ax + Bu \quad (27)$$

$$u(t) = -Kx(t) \quad (28)$$

For the assignment of the values of Q and R, it is convenient to calculate controllers based on different values for the matrices Q and R and verify their behavior. But an important point to take into account is that the values assigned to Q and R must be positive or zero. For this case, the following matrices are obtained corresponding to Q and R.

$$Q = \begin{bmatrix} 90000 & 0 \\ 0 & 90000 \end{bmatrix} \quad (29)$$

$$R = [1] \quad (30)$$

Results

The simulations of the rocker system with motor and propeller with the implemented controller are carried out with the SIMULINK tool integrated in the MATLAB software.

Obtaining the matrix K is the initial step for the integration of the LQR controller. Carrying out this task manually is complicated, so to obtain this matrix the function K = lqr (A, B, Q, R) is used, which is integrated into the MATLAB software. At this point, the necessary adjustments are made for the Q and R matrix until the desired behavior is obtained. The values obtained for matrices Q and R are indicated in the Equations (29) and (30), and matrices A and B are taken from the Equations (23) and (24). Therefore, the matrix K obtained is as follows.

$$K = [284.1221 \quad 297.8597] \quad (31)$$

The value obtained from matrix K is integrated as a gain into the system control diagram. This diagram can be seen in Figure 2, which already implements the LQR controller. It was developed in the SIMULINK tool.

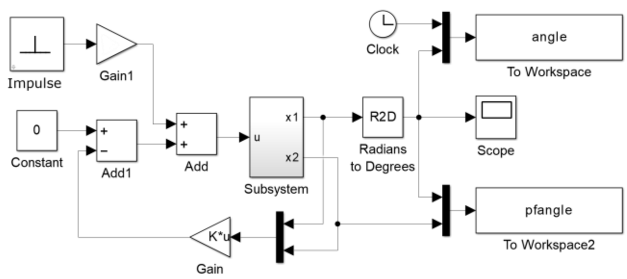

Figure 2 LQR control sysrem in SIMULINK
Source: Authors' own creation

Prior to the analysis of the response of the controller to disturbances of different magnitudes, the vector field corresponding to the solutions of the non-linear system that describes the rocker with motor and propeller is obtained. As can be seen in the vector field shown in Graph 1, the trajectories of the solutions of this system reach a value of zero, which indicates that the system is perfectly capable of reaching an adequate point of stability in the face of disturbances and after they elapse. a certain time. Furthermore, the Graph 1 allows the visualization of the relationship between the two states of this system, where X1 is the angular position (x-axis) and X2 is the angular velocity (y-axis).

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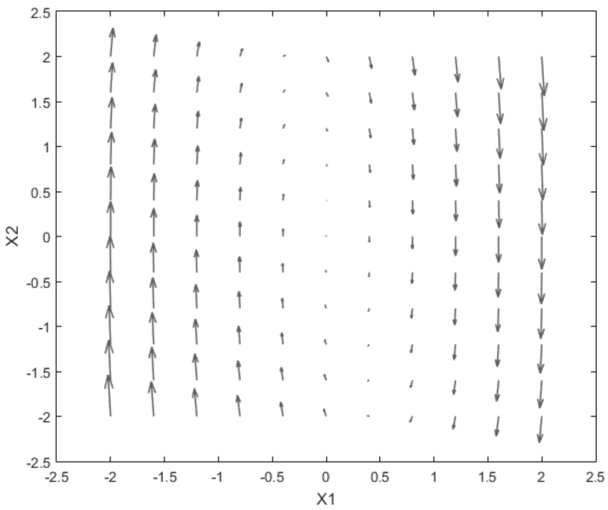
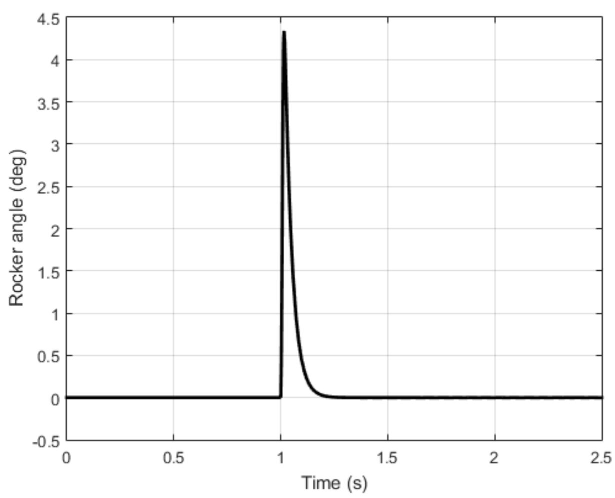


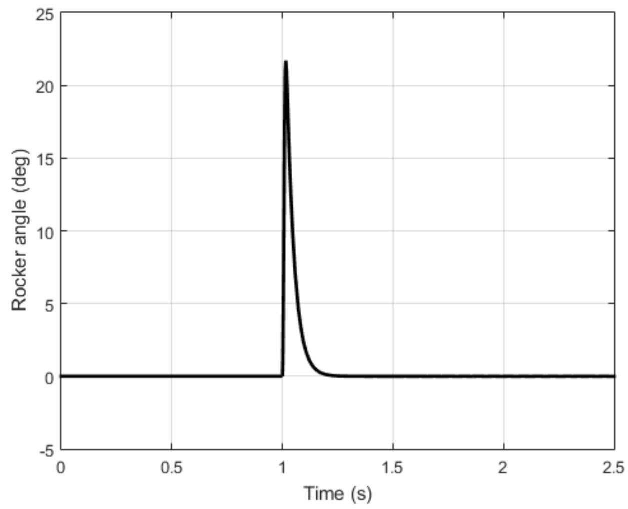
Graphic 1 Vector field of the system
Source: Authors' own creation

The simulations consist of applying two disturbances of different value and visualizing the response of the control to them. For the first case, a pulse disturbance is introduced with a reference at 0 ° and a magnitude of 1N. The response to this disturbance can be seen in Graph 1, from which it is identified that the rocker arm has an over-elongation of + 4.4 ° with respect to the horizontal, and a settling time of 0.3s.



Graphic 2 System response to a 1N disturbance
Source: Authors' own creation

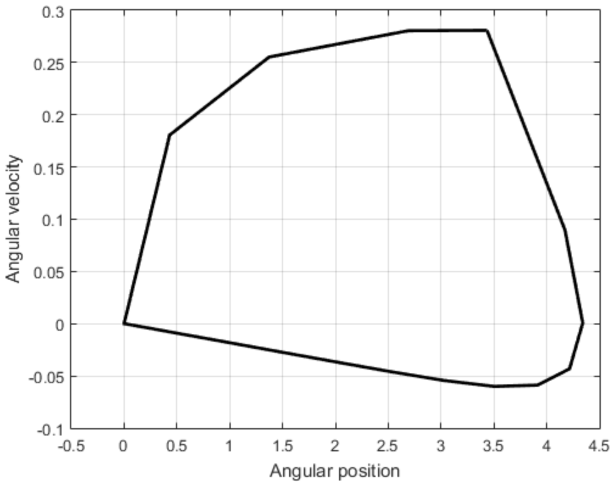
The Graphic 3 shows the response of the system to a disturbance with a magnitude of 5N and with a reference of 0 °. The response obtained for this disturbance indicates that the settlement time does not change with respect to the 1N disturbance since it is 0.3s, on the contrary, the over-elongation does present a change since it is + 22 °, change expected due to the increase in the magnitude of the disturbance.



Graphic 3 System response to a 5N disturbance
Source: Authors' own creation

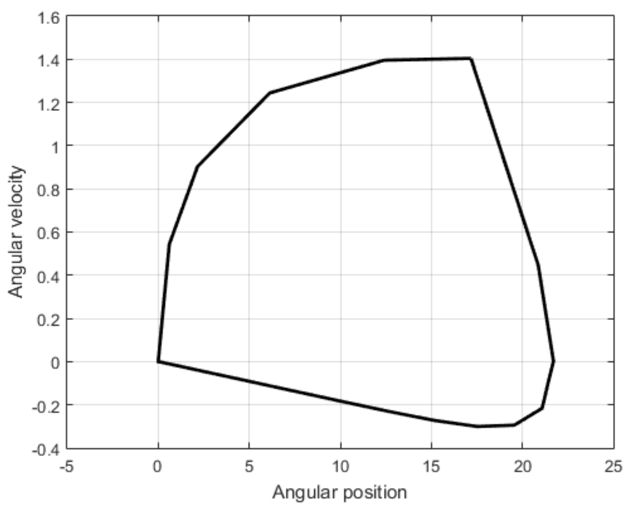
Subsequently, the stability of the control system against disturbances is analyzed. The stability is one of the most important characteristics during the development and subsequent analysis of a controller, and because of this it is the first point to consider for professionals focused on control. In the case of linear systems, there are some criteria that allow us to analyze the stability of the system, such as the Rout-Hurwitz and Nyquist criteria. However, in non-linear systems, the criteria mentioned are not applicable [19], so it is necessary to use other methods that allow stability analysis in non-linear systems.

To verify the stability of the control system developed for the rocker with motor and propeller, a graphic analysis is used through the use of phase planes, in which the variable X1 or angular position (on the X axis) is plotted against the variable X2 or angular velocity (on the Y axis). The Graph 4 shows the phase plane corresponding to the 1N disturbance, it shows that the trajectory of the system starts from the reference point determined in the initial conditions (which in this case is 0 °) and after a certain time elapses. reaches the reference point again.



Graphic 4 System phase plane for a 1N disturbance
Source: Authors' own creation

The phase plane obtained to visualize the stability of the system before the 5N disturbance is shown in the Graph 5 which, as in the previous case, starts from the established reference point and returns to it after a certain time, but for this case it can be observed that the values of the angular velocity and angular position undergo a change and this is due to the increase in the magnitude of the disturbance, but in both cases the trajectories converge in the same reference point.



Graphic 5 ystem phase plane for a 5N disturbance
Source: Authors' own creation

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Conclusions

In the present work, an optimal LQR controller is developed for a rocker system with motor and propeller, which is represented from a non-linear model, which allows obtaining the different variables that make the controller design possible. In addition, the dynamic behavior of the system is obtained in the face of disturbances of different magnitudes and having, in all cases, a reference point of zero in order to determine the over-elongations in relation to the angular position of the rocker. The responses obtained show that the system has a short settling time with a small over-elongation in the event of disturbances of small magnitudes and, if this magnitude increases, it has over-elongations of greater value with respect to those obtained for disturbances of less magnitude, but, in In the cases analyzed, the system reaches the desired value in a short time.

The stability analysis carried out from the phase planes indicates that the trajectory of the response of the system with the LQR controller, in the face of disturbances of different magnitudes, converges on the reference value determined in both cases. In addition to this, it is said that the controller developed for this non-linear system provides an appropriate dynamic performance and with precision of the angular position of the rocker, even having different perturbations.

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