RLC circuits: Obtaining the current graph using a mobile application

Circuitos RLC: Obtención de la gráfica de la corriente mediante una aplicación móvil

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Abstract

In this document we simulate the response (current value for t≥0) for a series RLC circuit through the solution of a second order differential equation. A mobile application is developed for the Android Operating System that obtains the graph of this current using the ANDROID STUDIO development environment and the GRAPHVIEW library. With the choice of the mathematical expression that defines the voltage in the circuit and the values of the resistance, the inductance, capacitance, the initial current and the initial voltage in the capacitor or initial charge, this graph is obtained, which when compared with what is obtained using a CAS, no difference is perceived, which shows the validity of the results of the app. This application can help engineering students to understand in an easy and simple way the concepts related to this type of circuits, thus promoting the use of mobile devices in teaching-learning environments.

Android Operative System, Differential Equation, Mobile application, Response of a circuit, RLC circuit in series

Resumen

En este documento simulamos la respuesta (valor de la corriente para $t \geq 0$) para un circuito RLC en serie a través de la solución de una ecuación diferencial de segundo orden. Se desarrolla una aplicación móvil para el Sistema Operativo Android que obtiene la gráfica de esta corriente utilizando el entorno de desarrollo ANDROID STUDIO y la librería GRAPHVIEW. Con la elección de la expresión matemática que define el voltaje en el circuito y los valores de la resistencia, la inductancia, capacitancia, la corriente inicial el voltaje inicial en el capacitor o la carga inicial, se obtiene esta gráfica, que al compararla con lo que se obtiene empleando un CAS no se percibe diferencia alguna, lo que muestra la validez de los resultados de la app. Esta aplicación puede ayudar a estudiantes de ingeniería a comprender de una manera fácil y sencilla los conceptos relacionados a este tipo de circuitos, fomentando así el uso de dispositivos móviles en entornos de enseñanza-aprendizaje.

Aplicación móvil, Ecuación Diferencial, Cirucito RLC en serie, Respuesta de un circuito, SO Android

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Introduction

The use of smartphones with the Android Operating System has increased mainly in aspects such as communications (messaging services) and social networks, which has made them contain increasingly powerful components (processor, memory, capacity, etc.), so they can be considered as small computers capable of performing scientific calculations, analyzing data, graphing results, among others.

The study of electrical circuits is of vital importance for electromechanical engineers, in renewable energies, among others, for which several resources have appeared for their study, among which are the use of simulators such as PSpice, DC / AC lab, edumedia, which also include more electronics elements, or more general simulators such as Matlab Simulink. For the use of these simulators it is necessary to have certain knowledge that goes from the concepts related to the electrical circuits to the approach of a differential equation and its incorporation in a certain language for the simulation by blocks.

The former also require an internet connection to use, while in the latter the license is usually expensive, in addition to requiring desktop computers to run. Other options to consider are the following, WolframAlpha, which can be used on desktop computers or mobile devices, PhotoMath (and similar as MalMath, Mathematical Calculator: do maths per photo). All these apps require an internet connection to operate, which generates costs for the use of data, while WolframAlpha can also generate a cost per download, in addition to the fact that they tend to generate resistance to its use due to the many functions they have incorporated.

For these reasons, the idea of developing mobile applications for the realization of scientific engineering calculations that do not require the use of the Internet and that are also easy for users to use. In particular, this document presents the development of a mobile application that graphs the response of a series RLC circuit, where the voltage is given by some mathematical expressions that the user must choose, and enter only the resistance values, the inductance, the capacitance, the initial current in the circuit as well as the value of the initial voltage in the capacitance.

The document is structured as follows. The first section establishes the physical and mathematical concepts that determine the differential equation approach for this circuit. In the second section the solution to the problem is determined for some mathematical expressions of the voltage. The document is structured as follows of the app is presented in the third section, so that in the fourth section the result shown by entering a data set in an example is shown. In the conclusions section, areas of opportunity for the improvement of the app presented here are mentioned.

RLC circuit

We begin by analyzing the differential equation of RLC circuits, also called second order circuits (Alexander y Sadiku, 2006).

A circuit consisting of a resistor R, an inductor L and a capacitor C and a fem (or voltage) V (t) connected in series, is called the RLC circuit, as shown in Figure 1.
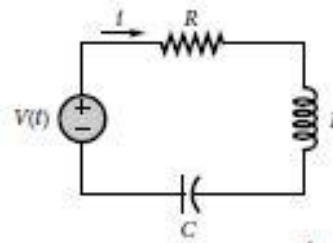

Figure 1 RLC circuit
Source: Wei-Chau, 2010

Kirchoff's voltage law (Wei-Chau, 2010) states that the algebraic sum of the voltages in a circuit is equal to zero, so if V_R, V_L and V_C are the voltages in the resistance, the inductor and the capacitor respectively then,

$$V_R + V_L + V_C = V(t). \tag{1}$$

If $i(t)$ represents the current in the circuit for $t \geq 0$, Ohm's law (Wei-Chau, 2010) states that,

$$V_R = Ri(t). \tag{2}$$

The relationship between inductance, current and voltage in the inductor is (Wei-Chau, 2010),

$$V_L = L\frac{di}{dt}. \tag{3}$$

Finally, the relationship between current and voltage in the capacitor is expressed by (Wei-Chau, 2010),

$$V_C = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau. \qquad (4)$$

From equations (2), (3) and (4) in (1), we get,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = V(t). \qquad (5)$$

When obtaining the derivative with respect to time in (5), we find the equation of the RLC circuits in series,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dV}{dt}, \qquad (6)$$

or,

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{L}\frac{dV}{dt}. \qquad (7)$$

Equation (7) must be complemented with the initial values $i(0)$ and $i'(0)$ in order to obtain the value of the current (response) for $t \geq 0$. In this way the problem to be solved is to obtain the graph of the current from the values R, L, C, of the mathematical expression that defines the voltage and of the values $i(0)$ and $i'(0)$.

In many of the texts of differential equations (Edwards & Penney, 2008, Trench, 2002 and Zill & Warren, 2015) an equation equivalent to (7) is obtained in which the load q (t) in the circuit is considered with its corresponding initial values $q(0)$ and $q'(0)$.

In the following section we obtain the solution of equation (7) for some expressions of the voltage as well as how to determine the value i '(0), which clarifies the approach used instead of the one mentioned in the previous paragraph.

Solution of the equation

To obtain the solution of (7) some expressions for the voltage are considered. For this, the numbers are defined (Alexander & Sadiku, 2006),

$$\alpha = \frac{R}{2L}, \qquad (8)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}. \qquad (9)$$

Its importance lies in the fact that the solution depends on an order relationship between them, as will be seen later.

The first situation to analyze is the case of a circuit without a source, that is, when V(t) = 0, for all $t \geq 0$. In this way, (7) becomes,

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0. \qquad (8)$$

If known $i(0) = I_0$ and the initial voltage V_0 on the capacitor, (5) concludes that (Alexander & Sadiku, 2006),

$$L \cdot i'(0) + RI_0 + V_0 = 0, \qquad (9)$$

or,

$$i'(0) = i1 = -\frac{1}{L}(RI_0 + V_0). \qquad (10)$$

In this way, the solution to the initial value problem (8) given the quantities $i(0)$ and $i'(0)$, where the latter is obtained from (10), it depends on the order relationship between (8) and (9), which is obtained from the solution of the characteristic equation (Trench, 2002) and described below.

If $\alpha > \omega_0$, It is said that the answer is overdamped, and is expressed by,

$$i(t) = \frac{m_2 I_0 - i1}{-2\sqrt{\alpha^2 - \omega_0^2}}\exp(m_1 t) +$$
$$\frac{i1 - m_1 I_0}{-2\sqrt{\alpha^2 - \omega_0^2}}\exp(m_2 t), \qquad (11)$$

where, $m_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, y $m_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$.

If $\alpha = \omega_0$, the answer is said to be critically damped, and is given by,

$$i(t) = I_0\exp(mt) + (i1 - mI_0)t\exp(mt), \qquad (12)$$

where, $m = -\alpha$.

Finally, if $\alpha < \omega_0$, It is said that the answer is undamped, and is given by,

$$i(t) = \exp(\beta_1 t) \cdot \left(\frac{i1 - \beta_1 I_0}{\beta_2}\sin\beta_2 t + I_0\cos\beta_2 t\right), \qquad (13)$$

where $\beta_1 = -\alpha$ y $\beta_2 = \sqrt{\omega_0^2 - \alpha^2}$.

For the general case, the amount $i'(0)$ can be determined from the knowledge of the values I_0 and the initial charge on the capacitor q_0 (Borrelli & Coleman, 1998), mediante (5),

$$L \cdot i'(0) + R I_0 + \frac{1}{C} \int_{-\infty}^{0} i(\tau) d\tau = V(0). \qquad (14)$$

Since,

$$q_0 = \int_{-\infty}^{0} i(\tau) d\tau, \qquad (15)$$

then, (14) becomes,

$$L \cdot i'(0) + R I_0 + \frac{q_0}{C} = V(0), \qquad (16)$$

from where,

$$i'(0) = \frac{V(0) - R I_0 - \frac{q_0}{C}}{L}. \qquad (17)$$

With the above, the requirements for the development of the app are determined in the following section.

App Development

For the development of the app, the cascade model is used (Somerville, 2005), in which the requirements are determined in one of the stages. The first necessary data are obtained from equation (7) in which the resistance (R), the inductance (L), the capacitance (C), the expression that defines the voltage, and the value appear $i(0) = I_0$.

The amount $i'(0)$ is also necessary to obtain the expression of the current $i(t)$. However, it can be obtained from the quantities on the right side of equations (10) or (17). Another of the necessary elements is an expression of the voltage, so the following were selected.

1. $V(t) = 0$.
2. $V(t) = A$, where A is a nonzero constant.
3. $V(t) = At$, where A is a nonzero constant.
4. $V(t) = A \exp(Bt)$, where A and B are nonzero constants.

This choice shows that up to two more data that the user will enter are also necessary, whereby the current i (t) is obtained in each of the aforementioned situations, under the considerations summarized in table 1.

Expression of $V(t)$	Cases to consider to obtain $i(t)$
$V(t) = 0$	a) $\alpha > \omega_0$. b) $\alpha = \omega_0$. c) $\alpha < \omega_0$.
$V(t) = A$	a) $\alpha > \omega_0$. b) $\alpha = \omega_0$. c) $\alpha < \omega_0$.
$V(t) = At$	a) $\alpha > \omega_0$. b) $\alpha = \omega_0$. c) $\alpha < \omega_0$.
$V(t) = A \exp(Bt)$	a) $\alpha > \omega_0$ y $B^2 + 2\alpha B + \omega_0^2 \neq 0$. b) $\alpha = \omega_0$ y $B^2 + 2\alpha B + \omega_0^2 \neq 0$. c) $\alpha < \omega_0$ y $B^2 + 2\alpha B + \omega_0^2 \neq 0$. d) $\alpha > \omega_0$ y $B = m_1$. e) $\alpha > \omega_0$ y $B = m_2$. f) $\alpha = \omega_0$ y $B = -\alpha$.

Table 1 Cases to consider for obtaining the response of the source RLC circuit: System analysis

With the analysis of the requirements, the following sequence scheme is performed that shows the interaction between the objects of the application, which is shown in Figure 2.
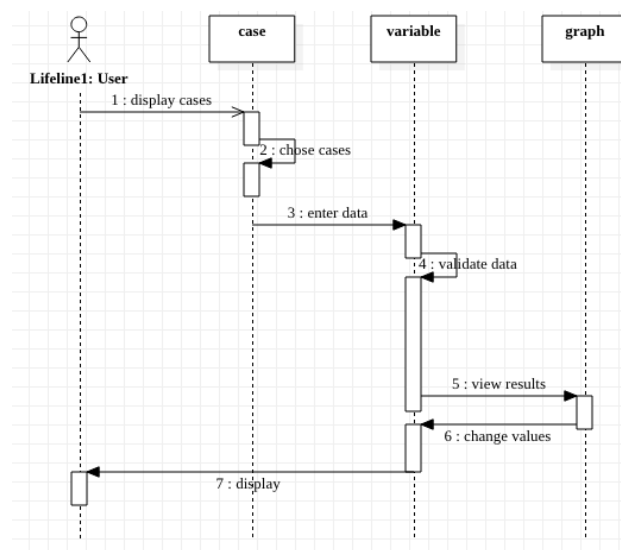

Figure 2 Sequence diagram of the app
Source: System analysis

The following section shows the graph generated by the app called RLCGraficador for a specific example.

Results

The example considered is taken from (Edwards & Penney, 2008). The values are considered $R = 16, L = 2, C = 0.02, i(0) = 0, q_0 = 5$ y $V(t) = 100$, which corresponds to the second line of table 1. The result obtained is shown in figure 3.

The comparison was made with that obtained by a CAS, which solves differential equations, finding that there is no difference between the two.

Figure 3 Graph generated by the app for the mentioned example
Source: System design

Acknowledgement

Conclusions

The development of mobile applications provides a way to streamline many tasks, particularly those involving numerical calculation and / or graphical representations of certain phenomena, since these save a lot of time that students spend to obtain the formal solution to practical problems.

For the design of the RLCGraficador app, it was considered to use a menu for the user to select the voltage expression and thus enter the necessary data. If by mistake any of them is not entered, it has the corresponding validations. With a button the user obtains the corresponding graph. If the user thinks so, he can obtain a screenshot of it, using one of the ways that most devices currently have.

When comparing the results obtained with the app and some CAS, it is noted that there is no difference between the two, which gives validity to the calculations made.

Among the aspects that can be improved are the following. The first of these would be the incorporation of more common expressions that define the voltage, for example trigonometric functions. This would allow the student to have more examples about how a given voltage determines the behavior of the current in an electrical circuit.

Another aspect of improvement would be to incorporate the generation of reports in .PDF format in which the user could visualize the data that was entered and the graph of the current, with which the app in question, it is not considered a pocket calculator.

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