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Knowledge diagnosis of trigonometric functions in engineering students

Diagnóstico del conocimiento de las funciones trigonométricas en estudiantes de ingeniería

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Abstract

Most students who begin university studies in Mexico do not have basic knowledge of mathematical objects, which causes students to fail due to poor performance. Therefore, the objective of this paper was to diagnose the prior knowledge of trigonometric sine and cosine functions in engineering students of a university in order to use this information in a didactic proposal that improves learning in such mathematical objects. For this purpose, an evaluation instrument was designed using the conceptual framework of Duval's semiotic representations and it was applied to a representative sample of 42 randomly chosen engineering students. It was found that the students have strengths in the cognitive activity of Treatment of the Graphic Register and in the Conversion of the Tabular Register to a graph. But they have deficiencies in the Treatment of the Tabular Register and in the Conversions from a Graphical to an Algebraic Register, and also from the Algebraic to the Tabular Register, both in the sine function, and only from the Tabular to the Algebraic in the case of the cosine function. It is recommended to enhance this knowledge in a teaching proposal to improve learning performance outcomes.

Diagnosis, Trigonometric functions, Semiotic representation

Resumen

La mayoría de los estudiantes que ingresan a las universidades en México no poseen conocimientos básicos de los objetos matemáticos, lo que provoca reprobación por bajo aprovechamiento. Por lo anterior el objetivo de este trabajo consistió en diagnosticar los conocimientos previos de las funciones trigonométricas seno y coseno en alumnos de ingeniería de una universidad, para utilizar esta información en una propuesta didáctica que mejore el aprendizaje en ese objeto matemático. Para ello se diseñó un instrumento de evaluación mediante el marco conceptual de las representaciones semióticas de Duval y se aplicó a una muestra representativa de 42 estudiantes de ingeniería elegida al azar. Se encontró que los alumnos tienen fortalezas en la actividad cognitiva de Tratamiento del registro gráfico y en la Conversión del registro tabular al gráfico. Pero tienen deficiencias en el Tratamiento del registro tabular y en las Conversiones del registro gráfico al algebraico, del registro algebraico al tabular, ambas en la función seno, y del tabular al algebraico en la función coseno. Se recomienda potenciar este conocimiento en una propuesta de enseñanza para mejorar los resultados de aprendizaje.

Diagnóstico, Funciones trigonométricas, Representación semiótica

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Introduction

Scientific and technological advances impose great challenges on higher education institutions, since they require training engineering professionals for an increasingly globalized and changing world of work. That is why in recent years there has been a growing interest in offering educational projects based on professional skills, with the intention of preparing professionals in a comprehensive manner, so that they apply their knowledge, skills, attitudes and values in the search for creative and innovative solutions to the problems of their professional practice (Martínez, 2014; Muñoz-La Rivera, Hermosilla, Delgadillo, & Echeverría, 2021).

Solving problems in engineering demands the use of mathematics, because it allows the data of any situation within a specific professional field to be modeled, analyzed and interpreted. This enables the possibility of discovering which paths to follow in order to reach solutions for a given problem. For this reason, mathematics is one of the basic sciences that are essential for the training of engineers (González, 2015; Sharhorodska, Padrón, & Bedregal, 2018).

Unfortunately, in Mexico it has been detected that there is poor mathematical training in students who graduate from high school. According to the last application of the PLANEA exam carried out in 2017, 66 out of 100 students reached only level I of proficiency, which represents the lowest level of the test. In addition, another 23 managed to reach level II. In other words, it was reported that 89 percent of high school graduates do not have a satisfactory command of the knowledge of their educational level at the time of finishing their studies (Secretaría de Educación Pública, s.f.).

This situation is probably the reason why not many high school graduates decide to study engineering. According to Quintero (2020), in a recent school year only 22% of university graduates were engineers. Currently there is a significant gap between the demand for this type of professionals by companies, and the supply offered by the country's universities. Demand exceeds supply, therefore there is a significant deficit of engineers in Mexico (Moreno, 2017; Magallanes, 2018).

On the other hand, the insufficient mathematical training of those who enter engineering careers also affects their school career to a certain degree. According to some studies, engineering students are the ones who fall behind the most during the first semesters of their training, due to the phenomenon of failure (Luna, 2020; Murillo-García, & Luna-Serrano, 2021).

Faced with this situation, universities have carried out some actions aimed at improving the progress of students in their school career. The reviewed literature shows that one of the strategies has been the development of studies that intend to understand the level of mathematical competences with which students enter university. The idea is to identify their strengths and weaknesses and take advantage of this knowledge to program teaching in a more suitable and effective way (Barrón, Ruiz, Luna, Estrada, & Loera, 2013; Minnaard, 2016; Carrasco, 2017; Gamboa, Castillo, & Hidalgo, 2019; Sánchez, Vidal, Molina, Reina, Guayasamín, & Pérez, 2019; Más, Sánchez, & Alegría, 2020).

It is within the scope of this line of research that the information reported herein is contained. The present research is related to the study of previous knowledge that students who enter an engineering career have on the subject of trigonometric functions, specifically the sine and cosine functions. The intention is to collect information that serves to program their teaching through experiences that contribute to meaningful learning instead of rote learning.

For diagnosis, the conceptual framework of Duval's semiotic representations (1998, 2010) is used, it establishes the conditions that must be met to consider that a mathematical object has been grasped. On the basis of the above, the objectives of this paper are:

General objective

Diagnose prior knowledge of the trigonometric sine and cosine functions in engineering students of a university, through the conceptual framework of Duval's semiotic representations, to use the information in a didactic proposal that improves the learning results of the students in such mathematical object.

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Specific objectives

a) Design a questionnaire based on the framework of Duval's semiotic representations, through a table of specifications, to determine the strengths and areas of opportunity of students around the mathematical concept of trigonometric functions.

b) Design a questionnaire based on the framework of Duval's semiotic representations, through a table of specifications, to determine the strengths and areas of opportunity of students in regard to the mathematical concept of trigonometric functions.

Justification

Functions are very important objects of study for mathematics and also for other sciences, since they are essential to develop certain concepts and serve as a model to explain a wide variety of phenomena. There are different types of functions and among these we find trigonometric functions. These have an important application in the phenomena that have to do with periodicity, which is why they are widely used in many sciences and engineering subjects. Trigonometry is a very old branch of mathematics and it was introduced in the 16th century as an essential tool for the development of mathematics (Cabrera, 2009).

In Mexico, the study of trigonometry begins from high school to university levels, so it is significant to identify the problems that occur at each school level, when teaching and learning concepts such as angle, conversions, trigonometric ratio, periodicity, among others; in order to improve the teaching and learning process of the trigonometric function concept (Montiel-Espinosa, & Cantoral, 2005).

In a study conducted with high school students, Aray, Guerrero, Montenegro, and Navarrete (2020) found that the difficulties students have when using trigonometric functions result from a lack of understanding of the basic concepts of this branch. They argue that at pre-university levels the training in trigonometry must be elementary and basic so that they can be used successfully at university levels.

As already mentioned, mathematics represents a learning problem for students entering university, and the academic unit where this work is carried out is no exception. According to reports from the academy in charge of teaching trigonometric functions, in the last school year, almost 50 percent of the students did not achieve the minimum desired performance, then, it is evident that some sort of intervention is needed to improve these results.

Consequently, the present research is justified, as its findings provide valuable information that can be used to identify areas of opportunity in the process of teaching and learning this mathematical object. Teaching can be organized to make learning more meaningful in the light of students' prior knowledge, thereby achieving better learning outcomes with lower failure rates.

By reducing failure, the transit of students in their school career is streamlined and thus contributes to reducing lag and improving the terminal efficiency of Educational Programs, indicators that affect many universities in the country.

Theoretical framework

Representation theory

Mathematical activity is in a very privileged place to analyze certain cognitive activities of the individual, such as conceptualization, problem solving, among others. In particular, the learning of mathematics requires the use of expression and representation systems for the cognitive functioning of thought, whether these systems are numerical systems, algebraic systems, Cartesian systems, or natural language. (Duval, 2010). Having access to and communicating mathematical objects relies heavily on their representations, so this research explores the use of Raymond Duval's semiotic representation registers, which are precisely framed in the phenomenon of the representation. Duval (1988, 2012) argues that mathematical objects are not clearly possible to perception, because of this a variety of semiotic representations of the object are necessary, but they should not be confused with the object itself, this is an imperative point for understanding math.

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Semiotic representations are those used to represent mathematical objects, they are constituted by the use of signs or symbols that belong to a representation system in which they have their own meanings and rules of operation. For example, a formula represents symbols or signs that are identified as an algebraic representation of the algebraic system, a graphical representation of the Cartesian system, a numerical table of the tabular system or a statement of natural language.

It is extremely important that there are diverse and varied semiotic representations of a mathematical object in order to interact, communicate and develop mathematical activity; likewise, the transformation of one semiotic representation into another that can be in the same register or from one register to another are essential activities. In this regard, Duval emphasizes that this second transformation is fundamental for the understanding of mathematical objects. According to Rojas (2012), transformations in the same register could also bring about various difficulties in the understanding of mathematics. Duval differentiates these two types of transformations as *treatments* and *conversions*.

Semiotic systems must be registers of representation to allow three fundamental cognitive activities associated with every representation, the first is the *formation* of a set of signs or symbols that are identifiable as a representation of something in a given system; for example, a formula y = 2 sin (3x+π) is identifiable in the algebraic log. The formation units and rules that are specific to each register are already given in the register and the important thing about this activity is to be able recognize them.

The second activity is *treatment*, which is the transformation of a representation in the same register in which it was formed, using only the rules specific to that register. For example, considering the algebraic register in the expression y = 2 sin (3x+π) if it is required to identify the "period" of the trigonometric function, certain algebraic manipulations must be carried out to find it, in other words, a treatment must be carried out algebraic with rules associated to that register.

Lastly, the third activity associated with semiotic representation registers is *conversion*, this is the transformation of one representation into another that belongs to another register, preserving all or only part of the content of the initial representation. According to Duval (1998) "The conversion is an external transformation to the starting register" (p. 276). For example, Figure 1 shows the algebraic expression of a trigonometric function and its conversion to the graph log. In the conversion activity you can keep all or only a part of the initial representation (algebraic representation).

Algebraic log ▼ Graphical log

$y = 2\, sen(3x + \pi)$

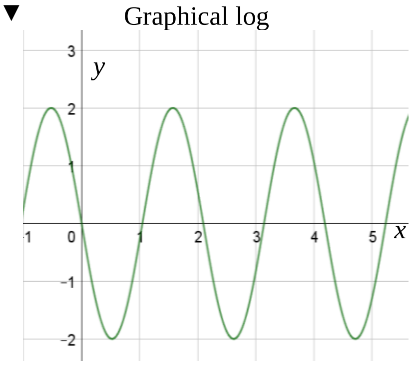

Figure 1 The algebraic and graphical representation of a trigonometric function
Source: Own elaboration

Converting from one register to another is often a difficult activity when there is no consistency between the registers. If there is congruence between the start and end representations, the conversion is trivial. A case of congruence in Figure 1, is the coefficient 2 that appears in the equation and in the graph, the same 2 can be seen on the Y axis which can be identified as the height of the wave; this is the element that corresponds to the term “amplitude” of the trigonometric function in both registers. But if we refer to the "period", it can be seen on the graph when the two waves are completed and it is measured as the horizontal distance from where the wave begins to where it begins again, but in the algebraic expression certain algebraic calculations are required to reach the value of the "period" which is P=2/3 π, in the light of the above, it is possible to exemplify the non-congruence between registers for the concept of "period".

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Therefore, the use of various registers to present the same concept is possible, as can be seen in the case of the "period", in this case the graph and the algebraic were used and there is no direct relationship between the two registers, so making a conversion between registers spontaneously will be difficult; hence, it is important to point out that the conversion activity needs to be taken into account in the teaching and learning process, starting from presenting specific tasks to the student (Duval, 1998).

Research methodology

This section describes the fundamental points of this work, such as: the type of research that was carried out, the subjects that took part in the study, the instrument used to collect information and the procedure followed to achieve the objective that was set in the inquiry.

Type of research

Based on the data collected from the participants, this report is of a quantitative nature, since the answers to a specific number of questions were numerical. But it is also cross-sectional, because the data was collected simultaneously from all the subjects at a specific moment of the inquiry.

Participants

42 randomly chosen students from engineering careers who were in the second semester at the Instituto Tecnológico de Sonora participated. Of these, nine were women and 33 men. The age of the participants fluctuated between 18 and 20 years.

Instrument

The questionnaire used to collect information (Table 1) was made up of nine multiple-choice items related to the trigonometric functions sine and cosine. Three of these were designed so that the students reflected the Treatment activity, one in the algebraic register, another in the graphic register and a third item in the tabular register. The other six items of the instrument were raised to observe if the students were able to carry out Conversions in both directions among the registers: graphic algebraic ↔, tabular ↔ algebraic and graphic ↔ tabular.

Procedure

The first activity was to develop the measuring instrument, with which the data of the investigation would be collected. This task was carried out based on the conceptual framework of Duval's semiotic representations (1998). In accordance with this theory, it was decided to include three semiotic representations of the trigonometric functions: the algebraic register, the graphic register and the tabular register, since they are the most common registers found in mathematical literature. To achieve validity of the content of the instrument, a table of specifications was used to relate the learning contents with Duval's cognitive activities according to the recommendations of Santibáñez (2011).

R	CA	Register Ini-Fin	Task
1	TR	A-A	Find the period of the function $y = 5sen\left(\frac{1}{2}t\right) + 1$
2	TR	G-G	Find the period and amplitude of the function in the graph.
3	TR	T-T	Find the amplitude and the period in a number table.
4	CO	A-G	Given the algebraic expression, identify from 4 graphs which one it corresponds to.
5	CO	G-A	The graphic of the trigonometric function is shown, and it is requested to identify which of the 4 equations shown is correct.
6	CO	T-A	It is requested to find A and B of the function $X(t) = Acos(Bt)$ in a numerical table.
7	CO	A-T	Given $T(t) = -8\,sen\left(\frac{\pi}{12}t\right) + 10$ the student is asked to complete the table and identify the maximun temperature.
8	CO	G-T	Starting from the graph of a trigonometric function and pointing out some points: A, B, C, D and E complete the table.
9	CO	T-G	Given three tables relate to their respective graphic.
Note: R=reactive; CA=cognitive activity; TR=treatment; CO=conversion; Ini=initial; Fin=end; A=algebraic; G=graphic; T=table			

Table 1 Task requested by each instrument item

The next step consisted of presenting the instrument to three experts in the teaching area so that they could validate that it evaluated what it was supposed to evaluate. Subsequently, the instrument was applied to a small group of students to rule out possible errors in their writing.

The next activity was to qualitatively classify the difficulty indices of the items, as recommended by Díaz and Leyva (2013). It was established that an item with an index greater than or equal to 0.9 would be considered easy. An item with an index from 0.8 to 0.89 would be a moderately easy item. From 0.7 to 0.79 the item would be of medium difficulty. From 0.5 to 0.69 it would be considered moderately difficult and less than 0.5 would be classified as a difficult item.

With the instrument already designed in its final version, a representative sample of engineering students was randomly selected in order to apply the measuring instrument to them at the initial moment of teaching trigonometric functions.

Finally, the responses of the students who made up the sample were coded, the difficulty indexes of each item were determined and they were classified qualitatively. With this activity, the difficulties and strengths of the students were identified, in terms of cognitive activities, so the objective of the research was achieved.

Results

42 students participated in the study, they answered nine items related to trigonometric functions, in the representation registers: tabular, algebraic and graphic. Three items corresponded to the cognitive activity of Treatment in each of the registers, and the remaining six items corresponded to the cognitive activity of Conversion of the three registers in both directions. The activity of the students was observed and their strengths and weaknesses were recorded.

Table 2 shows the average difficulty indices in the two cognitive activities that were evaluated with the instrument. The Conversion items had on average a smaller difficulty index than the Treatment items.

This means that the Conversions were more difficult for the students than the Treatments. This result seems reasonable in the light of the theory of semiotic representations. For Duval (1998, 2010), performing a Conversion is a task that demands a greater understanding of the mathematical object than the Treatment activity.

Cognitive activity	Successful items	Difficulty Index
Treatment	75	0.595
Conversion	131	0.520

Table 2 Average number of correct items and average index of difficulty by cognitive activity

Regarding the performance of the students in each item of the instrument, Table 3 shows that four of the nine items were difficult for the students, three were moderately difficult, one was of medium difficulty and another was classified as an easy item. These results show that students initially have little understanding of trigonometric functions, which confirms the low level of mathematical skills in students who graduate from high school and start university studies in Mexico. According to the Planea test applied by Secretaría de Educación Pública (s.f.), 89 percent of high school graduates in Mexico do not have a satisfactory command of mathematics.

Regarding the cognitive activity of Treatment, it was observed that the students showed strength in the graphic register and deficiency in the tabular register. On the other hand, in the cognitive activity of Conversion, their strengths were located only in the Conversion of the tabular register to the graph, but deficiencies were found in three Conversions: from the graphic register to the algebraic, from the algebraic to the tabular, both in the sine function and from tabular to algebraic in the cosine function.

R	CA	Register Ini-Fin	Difficulty index	Index classification
1	TR	A-A	0.57	MD
2	TR	G-G	0.79	DM
3	TR	T-T	0.43	D
4	CO	A-G	0.55	MD
5	CO	G-A	0.38	D
6	CO	T-A	0.40	D
7	CO	A-T	0.24	D
8	CO	G-T	0.62	MD
9	CO	T-G	0.90	E

Table 3 Index of difficulty of each item and its classification according to its value

Note: R=reactive; CA=cognitive activity; TR=treatment; CO=conversion; Ini=initial; Fin=end; A=algebraic; G=graphic; T=tabular; E=easy; ME= moderately easy; DM=medium difficulty; MD=moderately difficult; D=difficult

The fact that the areas of opportunity or improvement are observed more in the Conversions is possibly due to the fact that teachers do not commonly propose this type of cognitive activity in the learning tasks of their students, or also because the same bibliography used in teaching does not develop this type of activity deliberately. In a study carried out by Aponte (2016), it was found that textbooks do use different representations of trigonometric functions, but do not perform conversions from one representation to another, which hinders the understanding of this mathematical object.

However, Table 3 interestingly shows that item nine was the easiest item on the test. This consisted of a Conversion of the tabular register to the graph (Figure 2).

When reviewing the item in detail, it was found that apparently the students only verified the coincidence of the coordinates of a point in each table, with the coordinates of that point in one of the three graphs, which made it easier for them to distinguish which was the correct answer. Duval (1988) calls this congruence among registers, which means that the parameters of the original register (tabular) are easily observed in the final register (graphic). For this reason, this item should be modified in future applications of the instrument.

9. Match each table with its corresponding graph.

x	*y*
0	0
π	-2.8
2π	-4
3π	-2.8

Table 1

x	*y*
0	0
π	-4
2π	0
3π	4

Table 2

x	*y*
0	-4
π	0
2π	4
3π	0

Table 3

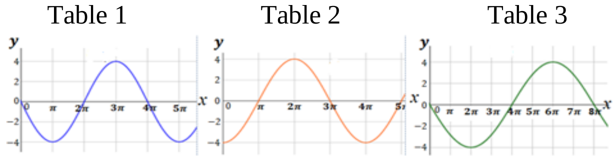

Table [ ] Table [ ] Table [ ]

Figure 2 Conversion item of the tabular register to the graphic register
Source: Own elaboration

On the other hand, the opposite occurred with the remaining Conversion items four through eight. For example, in item six (Figure 3), no consistency was recorded in the requested parameters of the initial register and the final register, which is why the Conversion activity requested in these items is not spontaneous and needs to be taken into account in the teaching process (Duval, 1998).

6. The following table shows the instantaneous positions of a mobile with oscillatory movement. Determine the values A and B in the function x =A Cos (B t) that models the following table:

t *(seconds)*	***x*** *(meters)*
0	-20
1	-10
2	0
3	10
4	20
5	10
6	0
7	-10
8	-20

A) A= 20, B = π/4
B) A = 10, B = 2π
C) A = 20, B = π/8
D) A = 10, B = 8

Figure 3 Conversion item from the tabular register to the algebraic register.
Source: Own elaboration

Finally, with the results obtained, it has become clear what strengths and areas of opportunity are in the teaching of trigonometric functions in the subjects studied. With this knowledge, as Agoiz (2019) mentions, teaching can be organized in order to enhance the previous knowledge that students have and thereby improve their academic performance. A strategy that is common in the teaching of mathematics and that has been used in various studies such as those carried out by Carrasco (2017), Sánchez et al. (2019) and Más et al. (2020).

According to the reviewed literature, it seems reasonable to recommend the use of some technological applications in the teaching of trigonometric functions. Studies carried out by some researchers report good learning results and conclude that the use of dynamic software such as GeoGebra or other similar ones is a great pedagogical aid, because they make it possible for students to generate different representations of trigonometric functions by manipulating some of their parameters. (Aristizábal-Zapata, Jiménez-Rojas, & Álvarez-Martínez, 2015; Molina-Toro, Villa-Ochoa, & Suárez, 2018; Vergara, 2021). Other studies such as the one carried out by Trípoli, Torroba, Devece, and Aquilano (2019), also point out the benefits of using physics laboratory equipment where an oscillating mobile is observed and measured, in addition to complementing learning tasks with the use of GeoGebra software. The idea is to start from something concrete, an observable phenomenon in the eyes of the students, and delve into the mathematical object as the software is used to dynamize the representations and thereby generate a deeper conception.

Conclusions

The general objective of the research was achieved, which consisted in the diagnosis of the previous knowledge that engineering students have of the mathematical object trigonometric functions.

It was identified that the students show strength in the cognitive activity of Treatment in the graphic register and Conversion from the tabular register to the graph.

On the other hand, the findings of this research show that the areas of opportunity in students are located in the cognitive activity of Treatment of the tabular register, and in the Conversions of the graphical to the algebraic register of the sine function, from the tabular to the algebraic of the cosine function and from the algebraic to the tabular of the sine function.

Therefore, it is recommended that the teaching of trigonometric functions should be programmed based on this prior knowledge that students have. The idea is to tackle the areas of opportunity that have been identified, without neglecting the cognitive activities where they show strengths.

In the same way and according to the reviewed literature, it is recommended to use a simulator or software such as GeoGebra as much as possible within the teaching-learning process, since it allows the student to manipulate the different representations of the trigonometric functions, which can contribute to a more comprehensive understanding and depth of the mathematical object. These and other innovations can be implemented in future research projects.

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