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A didactic sequence for the initial study of fractions

Una secuencia didáctica para el estudio inicial de las fracciones

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Abstract

The present study is focused on the mathematical object of fractions. A didactic support sequence was elaborated and implemented as the first formal approach to its study for students in the third year of elementary level school, using, as a means, accessible manipulative materials to be used in the classroom. In order to identify the theoretical elements that served as a guide for the design of the activities and the manipulative materials, analyzes were carried out from the epistemological and didactic approaches. The didactic sequence consists of three parts: the first is based on the previous notions that the student has about fractions, using their experiences with daily life; in the second part it is contextualized, trying to formalize the object, going from a middle to a half; Finally, the use of numbers to represent fractions is introduced, making use of figural representations. It was applied to 6 students. It was concluded that the process of building their knowledge about fractions, starting with ½, associating it with their experiences, and using manipulative materials, allowed students to outline the concept of fractions and arrive at its numerical representation, giving meaning to the numerator and denominator.

Didactic sequence, Mathematics, Elementary level

Resumen

El presente estudio está centrado en el objeto matemático de las fracciones. Se elaboró y se implementó una secuencia didáctica de apoyo como primer acercamiento formal a su estudio, para alumnos de tercer año de primaria, usando, como medio, materiales manipulables accesibles para utilizarse en las aulas. Para identificar los elementos teóricos que sirvieron como guía para el diseño de las actividades y los materiales manipulables, se llevaron a cabo análisis desde los enfoques epistemológico y didáctico. La secuencia didáctica consta de tres partes: la primera se basa en las nociones previas que tiene el alumno sobre fracciones, utilizando sus experiencias con la vida cotidiana; en la segunda parte se contextualiza, tratando de formalizar el objeto, pasando de la mitad a un medio; por último, se introduce al uso de números para representar las fracciones, haciendo uso de representaciones figurales. La secuencia se aplicó a 6 estudiantes, se concluyó que el proceso de ir construyendo su conocimiento sobre fracciones, iniciando con ½, asociándolo a sus experiencias y usando los materiales manipulables, les permitió a los alumnos esquematizar el concepto de fracción y llegar a su representación numérica, dando sentido al numerador y denominador.

Secuencia Didáctica, Matemáticas, Nivel primaria

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Introduction

The teaching and learning of mathematics is one of the greatest challenges in education, since, at the beginning of academic training, there is a predisposition to have conflicts with this subject, since sometimes, the way in which it is taught causes, throughout school life, a lag in the components of basic knowledge, which are considered a fundamental part in the construction of knowledge in mathematics (Gascón, 1994; Larios, 2005; Socas, 2011). In other words, they are like the foundations of a building.

In the process of learning mathematics, from basic to higher levels, there are many problems, in particular and one considered recurrent, is the difficulties in learning and teaching fractions; a problem that afflicts most students and teachers at all levels of education around the world (De León and Fuenlabrada, 1996).

From a very early age, the need to distribute things and the constant use of fractions begins, without taking into account the concept itself. For this reason, it is indisputable that this subject is in the basic education curriculum, as it is and will be present throughout both everyday life and school life. However, despite using fractions implicitly in everyday life, they constitute one of the greatest obstacles for many students, and their teaching constitutes a difficulty for teachers in basic education, extending this problem to higher levels of education (Block and Solares, 2001; Valdemoros, 2010; Cortina et al, 2012; Ávila, 2019; Arenas and Rodríguez 2021).

One reason that, according to researchers (Arenas and Rodríguez, 2021; Martínez and Lascano, 2001; and Block and Solares, 2001), give rise to difficulties in the learning and teaching of fractions is to be found in the nature of these numbers; that is, the origin of these difficulties can be explained by epistemology, given that this nature entails a diversity of meanings that the student has to link in order to form a global concept of these (Malet, 2010). Montrel (2022) mentions that handling the concept of fractions properly implies knowing and understanding in depth the concept of divisibility, which is why it is so important to work on fractions from an early age and in a more formal way at school.

A key moment to intervene in the learning of fractions is when students begin to study them, as it is considered that this is the right moment because it is when they begin to construct the first meanings of the concept of a fraction as a number, which was previously unknown to them. The importance given to this moment is supported by Freudenthal's (1983a) interpretation of what he says about attending to problematic cases of learning when they are detected at the beginning.

It is assumed that, if firm foundations of a content are not built, there are gaps in the construction of the concept, which then constitute obstacles and gaps in knowledge are formed, which eventually become a major problem at higher levels.

With regard to teachers, the authors Castro et al. (2015), Fazio and Siegler (2010) and Gairín (2001) agree in several articles that the meaning of fractions that teachers have is of vital importance, as it has a great influence on the construction of knowledge in their students. Pineda (2012) states that much depends on the teacher's conception of the concept, as well as the student's prior learning. Moreover, within primary education, in most cases, teachers are not specialised in the subject and, therefore, their teaching depends only on their experience as pupils.

Gairín (2001) carried out a study with trainee teachers on the representation systems of positive rational numbers where he analysed three things: learning models, representation systems and understanding of mathematical thinking. After conducting the research it was observed that student teachers tend to reproduce their knowledge as they learned it.

In the process of learning fractions, teachers play a major role. If the teacher has wrong knowledge about fractions or does not reflect on his or her conception of the subject, in most cases he or she passes on these deficiencies to the students, which is one of the difficulties in teaching and an obstacle to learning.

Another obstacle that arises in the learning of fractions derives from the previous one, because the teachers' knowledge is also decisive when it comes to organising the activities to be carried out in the classroom. Brousseau (2000) mentions that:

Nowadays the term didactics comprises the activity of teaching mathematics itself, the art and knowledge of doing it, the ability to prepare and develop the resources to carry out this activity and everything that is manifested around it and in addition to asking the teacher to choose problems that provoke learning in the student that manifests itself through new answers (p.33).

Block et al. (1991) discusses the role of problem solving in the classroom, its uses and what teachers think of it. He comments that in most cases teachers propose a problem and leave similar problems for their students to solve. In this work Block invited the teachers not to use routine exercises, but to propose new exercises to their students, to which the teachers were sceptical, as they think that they will not be able to solve them and if they try, they are very attentive to guide them, marking the error or simply giving them clues for their resolution without leaving them entirely to their own devices to solve them, something that truncates the development of students' skills, as it is by making these mistakes that they can learn.

On the other hand, the difficulties in learning presented by the students could be attributed, in a certain way, to the fear they have of mathematics and even more so to the subject of fractions, which, just by seeing them, complicates the whole exercise, since, from the beginning, they do not construct a good meaning of these.

With regard to the concept of fractions, Fazio and Siegler (2010) consider that the difficulties in learning fractions are the result of a lack of conceptual understanding, derived from the different situations to which they are linked and the different ways of representing them. For this reason, some research (Malet, 2010; Rios, 2019; Cortina et al., 2012) highlights the importance of addressing the different meanings, and from the outset emphasising the clarity of each one of them in order to be able to relate them as a whole and not see them as totally different scenarios.

González and Block (2005), explore the didactic potential for learning the notion of fraction from the division of a unit fraction by an integer, due to the difficulties presented by the students in solving the didactic situations, it cannot be concluded that the use of the fraction in this way can favor the understanding of the notion of the fraction in the division algorithm.

According to these analyses of research on the difficulties of fractions, the factors that could lead to these difficulties are evident, such as: lack of conceptual understanding of fractions, teachers' lack of conceptual understanding of fractions in the learning process, and the lack of conceptual understanding of fractions in the learning process. For this reason, it is important for students, from their first approach to the concept, to construct and make sense of the different characteristics of fractions through reflection.

Target

Develop a didactic sequence that leads students to schematise the concept of fractions, making use of manipulative material and different contexts, allowing them to interact with each other and have references so that they can identify fractions in their graphic and numerical representation in order to give them a meaning when they begin to study them.

Conceptual framework

Epistemological analysis

The epistemological analysis seeks to clarify the origin and nature of fractions; what are fractions, their characteristics, order relations and their different meanings.

Since ancient times, man has had the need to quantify things in a variety of situations that arise, comparing and ordering different sets, for example: sheep in a flock, people in a group, bottles of milk on a shelf and cakes in a bakery, establishing equivalences between these sets and assigning representatives to each class of equivalent sets, which are now the natural numbers. Out of this, relationships such as less than, greater than and operations such as addition and multiplication emerge, which are used as tools to solve problems.

However, although these numbers can be used to model a wide variety of situations in which a collection of separate or discrete objects must be quantified, problems arise in the field of measurement when trying to use natural numbers, as there are situations in which they are not suitable or sufficient for modelling, for example, the case of determining the weight of a sheep, the height of a person, the amount of milk in a bottle and the slice of a cake (Skemp, 1999).

The way of quantifying objects that are not discrete makes it necessary to cut the given object into equal parts until one part is small enough to serve as a unit of reference and when combining some parts they match the given object (Skemp, 1999). This process of quantification, which makes use of slices of objects and their combinations for measurement purposes, has as a product what has been called a fraction: "each of the separate parts of a whole or considered as separate" (Real Academia Española, n.d., definition 2).

The notion of fraction makes sense in measurement situations through the operations of cutting and combining. After a long period, it was decided to separate this symbol from the relationship linked to measurement processes and quantities of measurement; to consider it "a number, an entity in itself, on the same plane as the natural numbers. When a and b are natural numbers, the symbol is called a rational number" (Courant, 1979, p. 61). This identification of fractions as a number is justifiable by the mathematical process of defining a relation between fractions (namely, $a/b \sim c/d$ if $a*d = b*c$) which turns out to be one of equivalence and which produces a partition of these, which is seen as a subset of a formal number system, that of rational numbers, so that the representatives of the classes in the partition constitute the set of fractional numbers.

Intuitively, the identification of these classes of fractions as rational numbers can be seen as bundles or boxes of equivalent fractions. Each box is a rational number, so that the rational represents the fractions and vice versa (Pujadas & Eguiluz, 2006).

Moreover, addition and multiplication of rational numbers follow, among others, the same laws that govern natural numbers, the commutative and associative law of addition, the commutative and associative law of multiplication and the distributive law of the product over the sum. All this leads one to believe that the relations and operations of the natural numbers are extended to the fractional numbers, however, the latter turn out not to be similar nor to have the same meanings.

Courant (1979), explains that although fractions are closely related to natural numbers, being an extension of these, when performing operations such as addition or multiplication, students have difficulties, as they face rules or meanings different from those they are used to with natural numbers, given that, as mentioned before, fractions have different algebraic structures, they have their own essence. He also mentions that rational numbers are their own creation, and that the rules depend on the will.

For example, the rule of addition of the natural numbers could have been extended to fractions, but in calculating the result it would be absurd, since if one adds:

$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$

The result does not coincide with the meaning of the sum, since we would be obtaining a quantity equal to one of the addends.

In this sense, something similar happens with multiplication, only that in this case the rule is the same, what changes is the interpretation of multiplication in the natural numbers, where when multiplying a and b natural numbers, the result is greater than these numbers, on the other hand, with fractions, it is different, when multiplying,

$\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

the result obtained is a smaller number.

On the other hand, while for natural numbers there is only one type of number, for fractions there are different types of fractions: proper fractions, improper fractions, mixed fractions and unit fractions.

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In addition to the differences in the structure and operations with fractions, fractions are used in different contexts, which leads to different meanings for the fraction, adding further complexity to the construction of the fraction concept.

Different meanings of fractions

Moving from the understanding of the concept of natural number to the understanding of the concept of fraction implies a great leap, which goes much further than just following the rules and structure of natural numbers, as the second concept is not a simple extension of the first one. According to Vasco (1994, cited by Meza and Barrios 2010), a reconceptualisation of the units of reference and the process of measuring itself is required. This is due to the different characteristics that fractions have with respect to natural numbers, ranging from their genesis, written representation, types of fractions and interpretation of operations.

In this sense, the fraction can be interpreted in various ways, i.e., it has different meanings depending on the context in which it is used. Thus, the fraction as a mathematical object is considered, according to Kieren (1976), as a mega-concept, since it is seen as a complex synthesis of the different meanings associated with it.

The importance of these different meanings lies in a way in that they break down the possible uses given to fractions in the different contexts of everyday life, contexts that provide meaning to fractions beyond their formal definition as a pair of natural numbers or as a solution of an integer equation of the form $bx=a$ ($b\neq 0$).

Kieren (1976) identifies the different meanings that can be associated with a fraction as: part-whole, quotient, ratio, operator and measure, meanings which are described below.

The fraction as a part-whole

This meaning of the fraction arises in the context of relating the part constituted by a certain number of equal fragments into which the whole has been divided for comparison purposes (Kieren, 1980).

In such a case the relation answers the question what part is of the whole, and the fraction becomes a representative of this relation, in which the given number of fragments that combine to constitute the part and the number of equal parts into which the whole needs to be cut are used.

The fraction as a quotient

The fraction, under this meaning, is associated with the operation of division between two numbers. As a quotient, it answers the question: how much does each one get? Moreover, this meaning, in real situations, gives rise to equal sharing, "The most general representation of fraction in the form a/b leads to the immediate idea of quotient of two numbers: a units in b equal parts, with which appears the notion of sharing in equal quantities" (López, 2012, p.15).

The fraction as a ratio

The context in which the fraction acquires the meaning known as a ratio is when, when comparing two given quantities, it is possible to find two whole numbers a and b whose ratio is equal to the ratio of the quantities, in which case, this ratio can be represented as a/b. This type of meaning is associated with real-life contexts such as scales, proportionality and percentage (Bressan, A. & Yaksich, F. 2001). These authors also mention as an example that in the context of percentage when talking about mixtures, a relation of quantities is established, such as in the case of a solution with 4% of sodium bicarbonate and this is represented in relation to a whole as 4/100.

The fraction as an operator

In this meaning, according to Kieren (1980), the fraction acts as a multiplicative transformer on a set, magnitude or number, which implies a reduction or enlargement on the set to which the fraction is being applied as an operator.

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The fraction as a measure

Fraction as a measure, according to Kieren (1980), arises in attempts to quantify a region that lead to the assignment of a fraction as its magnitude. As part of such attempts, this might start by taking a magnitude as a unit of reference and observing what is the largest number of times that unit fits in the region to be quantified and, if necessary, resorting to a process of dividing the unit into equal parts to try to cover the missing part of the region, until this is achieved. In the end, summarising the process, a fraction (a/b) will be arrived at which quantifies the number of times it is necessary to take the selected reference unit to cover the given region.

Differences in both the structure and the use of fractions in different contexts give rise to different meanings, which are one of the main conflicts that lead to problems for students and teachers in learning and teaching fractions.

Methodology

In this work, a sequence of activities based on the use of manipulative material was created with the aim of allowing the student to use it to corroborate the certainty of what he/she might think is the solution to the problem, or to use this manipulation to propose his/her answer. In addition, this proposal provides the teacher with a guide, seeking to support him/her in directing the activity, and to give him/her ideas on how to relate the contents of the sequence to contexts, so that the students can become interested in solving the problems posed.

Participants and curricular context

The sequence was carried out in two phases: the first phase was aimed at finding out what the teachers thought about putting it into practice, as well as how the students reacted to working with manipulatives. This first phase was carried out in three groups of third grade of primary school with about 25 pupils (8-9 years old) each, in each group there was one teacher. This school year was chosen because this is when the pupils first approached the mathematical object of fractions. It was the teachers who were in charge of guiding the didactic sequence with their pupils, which allowed them to be observers taking notes on how the activity was carried out.

We worked with a preliminary design, which, according to the observations obtained, allowed us to redesign the sequence and thus put it into practice in a second phase. In this second phase, we worked with a group of 6 third grade students, who had no theoretical (school) knowledge of fractions. This time, they had the opportunity to take on the role of the teacher in applying the sequence of activities.

First phase: preliminary design of the sequence

Taking the epistemological and didactic analyses as a reference, it was decided to focus the proposal on the first approach that primary school students have to the study of fractions. One of the strongest motivations for placing the emphasis on this part of the thematic content was derived from taking into account what Freudenthal (1983b) said and ratified by multiple investigations about the importance of giving adequate meanings to mathematical objects from the beginning.

The initial selection and analysis of the conceptual references allowed us to take them as a practical guide of principles to start the design of the activities that make up the Parts of a Whole Sequence, and at the same time, to have a concrete reference for their subsequent evaluation or analysis.

In this staging, something important to mention is that the students already had notions of fractions, so, by doing the sequence of activities with the manipulative material, the aim was for them to reflect on what they had already seen and begin to make sense of the fractions.

Before starting with the staging, we talked with the teachers about what we wanted to achieve with the sequence and we gave them a document where we provided guidelines for each activity, so that each teacher could guide the activity, in order to achieve the objectives set.

In addition to the observation, a series of questions were asked to the teachers, as it was considered important to take into account their experiences working with the students, as well as to have their opinion about this type of activities in the classroom.

In general, the three teachers were very enthusiastic about the use of this type of activities to complement the topics marked in the syllabus, they commented that the wording of some questions was confusing and long for the students, they felt that the sequence of activities was a bit long to put into practice in a single hour of classes, And as far as the manipulative material is concerned, they commented that it was very satisfying to see the students enthusiastic about doing the activities, as these materials helped the students to achieve a better understanding and to be able to solve the activities by themselves, thanks to this, one of them commented that very few students approached her to ask her any questions or what to do.

As the students already knew about fractions, they had already done the instructions in their textbook, the results and the way they worked was not what was expected, because although the material was attractive to them, the premature use of symbolic representations for these numbers is visible, as well as the lack of specific guidelines to clarify the purposes of the questions in each activity and suggestions of contexts that allow the teacher to intervene to promote a link between the experiences of the students and the questions presented in the material.

Based on the analyses carried out in the staging of the preliminary design, a redesign of the sequence of activities was undertaken, incorporating the observations made by the teachers. Against this background, it was decided to rework the original purposes of the sequence design. The final version of the sequence was as follows.

Activity 1. Paper folding of the sequence, is sectioned with the intention that the student builds step by step the characteristics of this first fraction that is being studied $\left(\frac{n}{2}\right)$.

- **Section I. Manipulation: half**. It focuses on the study of the fraction 1/2. Its purpose is for the pupils to begin studying fractions based on the previous notions they have from their everyday life experiences, i.e. at first they are expected to use their knowledge of half, a fraction that pupils handle naturally from an early age. This is done by folding a manipulative circle (see figure 1) so that, through its use in the different tasks, pupils have their first school contact with this fraction.

- Questions are posed that gradually lead students to discover the characteristics of this fraction, such as: identifying that a whole has two halves, as well as that the parts must be equal. In addition, guidelines are given to the teacher in order to give him/her ideas on how to guide the activity so that the student can make analogies between his/her experiences with the half and the activities that the student is asked to carry out, so that the activity flows in an appropriate way and the objectives are fulfilled.

Figure 1 Material used in activity 1

- **Section II. Context: from the middle to a medium**. Students are asked to take actions with this same fraction (a half) in an everyday context, such as buying a kilo or a half kilo of tortillas, with the aim of relating the notion of half to that of a half. In this part, the teacher's guidance is very important, as it will consist of promoting the students' reflection through questions such as: how many halves does an integer have? and how many halves does an integer have? So that they realise that they represent the same quantity and begin to restructure the notion of this fraction. Once they have evidence of fluent use of the fraction by giving appropriate answers to the questions posed, they move on to the next section.

Section III. Introduction to the use of numbers to represent means. It begins with the identification of their numerical representation, still linked to figurative representations (verbalising, but using numbers for this) so that, finally, they manage to approach, with an appropriate meaning, the writing of the fraction one half, associate the fraction two halves with the unit and a first approach to improper fractions with numbers greater than the unit (all referring to the denominator 2).

In this section, a box is added in which the child is shown the numerical representation of the fraction, one half, 1/2, describing what these numbers represent. It is suggested to the teacher that it is read as a whole so that, if necessary, clarifications can be made, as it is considered as a local institutionalisation in his or her charge.

Section IV. Writing proficiency. Students are asked to write down the fraction that certain figures represent, to relate the figurative part to the numerical part and to write the names of the fractions that these represent, all this using different figurative representations and units of reference larger than the unit.

With what has been done in activity 1, the aim is for students to be able to relate the half to its equivalent in mathematical language, the half, both in verbal and numerical language.

Activity 2. Cutting out paper. In this activity, which consists of two sections, the aim is for students to outline the space covered by the parts of fractions, halves, quarters and eighths. In addition, they should be able to make sense of the names of the fractions, construct graphic representations of them, make comparisons between them through graphic representation, construct the numerical representation of halves, quarters and eighths (see figure 2), and make comparisons between them by means of graphic representation.

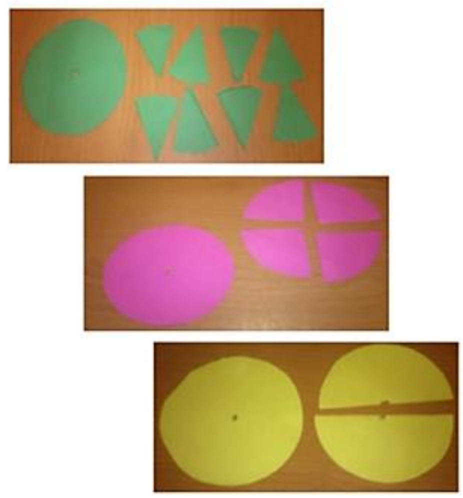

Figure 2 Material used in the activity 2

- **Section I. Naming fractions**, The aim of this activity is to give continuity to the relationships established in the previous activity in order to establish a strategy to obtain quarters and eighths by dividing in half each of the parts obtained previously (having started with half of the circle). The name given to the fractions is also determined, associating the number of parts contained in the unit with the ordinal name they already know (fourths, fifths, sixths, sevenths, eighths); although several are shown, emphasis is only placed on halves, fourths and eighths.

It is noted that the manipulation of the circles by cutting out the requested parts is intended to provoke the relationship of the requested fraction with the space it covers; that is, it provides a different experience from the mere visual perception provided by the fixed parts in a drawing, since these cut-out parts can be superimposed on the reference circle and make their relative size palpable depending on whether they are equal denominators (the sizes are related to the numerators), or different (these denominators are related in unit fractions).

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Section II. Identifying and comparing fractions. Manipulation makes it possible to isolate parts and compare them; not only to see which is larger, but also to determine how many times one fits into another (how many eighths cover certain quarters, or how many of these cover certain other halves, etc.). These manipulative comparisons are associated with the corresponding writing of the fractions represented, so that they can make sense of the order of the fractions and the role played by the denominator of the fraction in these orderings and comparisons. Of course, questions about the equality of fractions (matching their size as parts of the reference circle) written with different numerals are also included.

Activity 3. Jigsaw puzzles: in this activity the aim is to extend the previous notions recently acquired about halves, quarters and eighths, but taking a reference that could bring them closer to the handling of discrete objects, in this case 40 square pieces of a rectangular jigsaw puzzle (see figure 3). They are also asked to compare fractions using the same numerator and quarters and eighths as denominators, with the aim of identifying the differences between the order relations of naturals and fractions. In addition, three items are proposed as an assessment of the knowledge acquired by the students, in which order relation schemes and graphical and numerical representations are put into play (using as a reference unit, units larger than the unit).

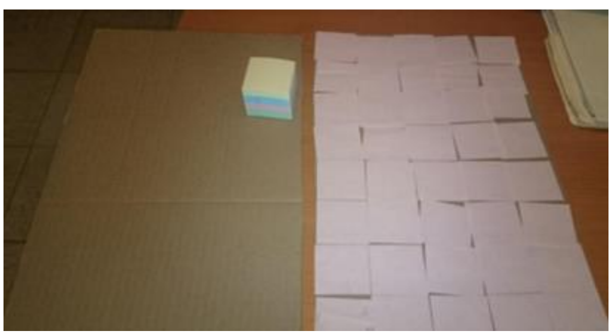

Figure 3 Material used in the Activity 3

- **Section I. Constructing halves, fourths and eighths, with discrete objects** the aim is to associate a half and a quarter with the number of pieces they represent of the total, using this rectangle as a framework so that the pupils can use the figural partitioning they have already mastered for these fractions.

In this context, they are asked questions that lead them to count pieces for different partitions; they are also asked to make comparisons between the number of squares that make up the different fractions that represent them, so that they become familiar with this type of partition by carrying out counting strategies that finally lead them to handle the numerical expression of the fraction they already know.

- **Section II. Comparing fractions with the same numerator.** Comparisons are asked to be made between fractions with the same numerator using as a medium the squares and the base; placing and removing squares from the base (see figure 4), to form the fractions and identify which is bigger or smaller, according to the number of squares associated to the fractions to be compared. Finally, three questions are presented, one of which asks students to relate the figurative representation to the numerical representation and, based on these, to put the fractions in order. The last test shows a series of figures in which different combinations are made to obtain halves, quarters and eighths, with the aim that the student is not left with only one way of dividing the figures.

Figure 4 Forming parts from discrete material

On the other hand, as a product of the previous interventions that were made in a first staging, a teacher's guide was incorporated into the design, since it became clear in those experiences that the teachers in charge of conducting the sequence required precise orientations to continuously clarify both the specific purposes of some of the proposed actions, to know what to expect as an adequate response or how to guide the students without directly giving them the answers or carrying out the procedures to achieve them.

In order for these clarifications, orientations and suggestions for the teacher to be integrated into the actions requested of the learners in each activity, they were included in a two-column table format (see figure 5), so that the text for the teacher would be next to the text for the learners, right next to the paragraphs referred to. On the other hand, as a product of the previous interventions that were made in a first staging, a teacher's guide was incorporated into the design, since it became clear in those experiences that the teachers in charge of conducting the sequence required precise orientations to continuously clarify both the specific purposes of some of the proposed actions, to know what to expect as an adequate response or how to guide the students without directly giving them the answers or carrying out the procedures to achieve them.

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<table>
<tr><td colspan="2">Activity 1: parts of a whole

Materials:
- 4 colored paper circles of the same diameter (2 white, 1 green circle | cut into two different parts of half and 1 pink circle cut into halves).
- Colors.
- Scissors.
- Pencil.
- Worksheet.

Part one: parts of a whole

1. Manipulation.</td></tr>
<tr><td>Instructions:

1. Take the white circle, folded into two equal parts and color one of the parts yellow:

A) What part of the circle is colored yellow?
____

B) How many halves does the circle have?
____

C) Which is bigger one half or two halves?
____________</td><td>Guidance for the teacher:

It is recommended that you use the manipulative material so that the children become familiar with the word half and can begin to use it in the meaning of part whole. In addition, the partition can be linked to the cast through verbalized questions. For example:

- Imagine that the circle is</td></tr>
</table>

Figure 5 Example of the format of the sequence of activities

Staging the redesign

This staging was carried out in order to observe whether the redesign of the sequence met the stated objectives. The implementation of the sequence of activities was carried out in a public primary school with a group of 6 third grade students, who had no theoretical (school) knowledge of fractions.

The sequence, as mentioned above, consists of three activities, which were carried out one per day in the classroom, each lasting approximately one hour.

At the beginning of the first activity of paper folding, the need to use the experiences that the student has in order to start with the construction of knowledge, as mentioned in the principles of constructivism, as these bases help the student to become familiar with and give rise to new knowledge. In this activity, it was necessary for the teacher to intervene in order to provide the students with contexts related to the questions on the worksheet, so that they could answer, because despite using manipulative material at the beginning, it did not tell them anything by itself, so it was necessary to relate, for example, the parts of the circle with chocolates and put them in a context of distribution.

In the second section of this paper folding activity, the students were asked to work in teams (pairs), so the teacher's intervention was important to be aware of the interaction between the students. As this was the first attempt in which the students would change their scheme from the word half to a half, it was necessary to be aware of what each pair of students was saying about the use of a half or a half. This intervention consisted of guiding the students through appropriate questions related to contexts in which they had a notion of this word according to their experiences.

In activity 2, Cutting out paper, the teacher's intervention was important to institutionalize the name of the fractions, associating the number of the parts with the corresponding ordinal number. In this activity, the manipulation helped the students to create a scheme of how to construct the numerical representation of different fractions.

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The last activity, Puzzles, although closely linked to the figurative part that the students had been working on, caused a bit of conflict, partly because of the change of manipulatives, the material they were made of and the context used. However, this could be rescued by placing the pupils in a context within their reach: for example, by asking them to imagine marbles instead of the puzzle pieces and to make piles of them (see figure 6).

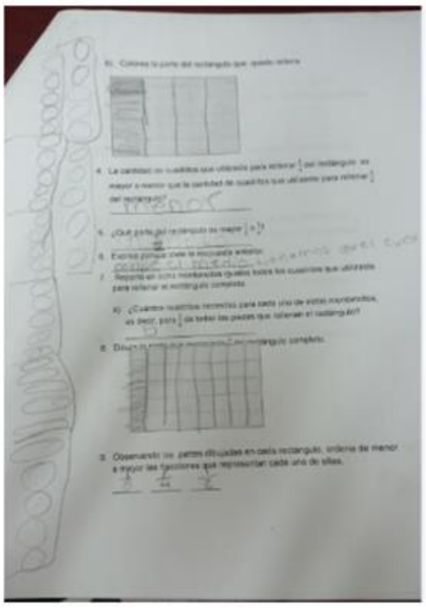

Figure 6 Construction of an octave using discrete objects

Throughout the staging, the importance of the guidance given in the teacher's guide was corroborated, so that the teacher could make use of them. In general, in the interventions that were made, they were used, giving the pupils more ideas on how to relate their knowledge to contexts and this made the pupils interested in learning about fractions. In addition, the redesign of the sequence of activities based on the use of short questions and pictures was very useful, as the students were not overwhelmed with reading and could follow the instructions quickly.

As far as the use of the manipulatives is concerned, there was a lot of enthusiasm on the part of the students to do what was asked of them in each instruction. It was very useful for them to be manipulating the circles, folding, cutting out and colouring, in the second activity Cutting out paper (see figure 7) the cut out parts were useful for them to make comparisons, and this in turn, to start with the recognition of the spaces to which the different fractions correspond, as well as to create an idea of the meaning of denominator.

Figure 7 Use of cut-out material

In Puzzle activity 3, the manipulative was not very useful due to the material it was made of, as it was very difficult to handle. Perhaps if the squares had been made of wood or some other rigid material, it could have been better manipulated and thus avoided some of the conflict that arose when trying to represent parts of the rectangular base by putting the squares on and taking them off. However, part of the objective of using manipulative materials was to make it accessible to the pupils, and if it were made of another type of material, the objective would be lost. The activity could be rescued by making use of drawings that allowed the students to do what was asked of them in a satisfactory way.

In general, the material was very supportive. It was observed that by doing the manipulations of the material necessary to answer the questions; cutting out, colouring and counting, the pupils are entertained and do it with pleasure. Most of the time, when reading the questions, they used the circles or parts of the circles and counted the parts to answer. In addition, they were also useful for the teacher, because sometimes the students asked for help and the teacher manipulated these materials so that they could observe and look for the answer by themselves.

In the staging of the activity, the importance of the previous experiences of the student emerged in order to start the construction of new learning, as stated in the constructivist approach (Chadwick, 2001), since it was the first time that the students were introduced to the use of fractions in the classroom, the teacher had to make an association between a context of distribution and what was described in the first activity, using chocolates, in order to make sense of this first encounter between fractions and students.

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Also in this staging it turned out that, far from what one might think, the role of the teachers is of vital importance, as their work consists of leading, through questioning or changing contexts supported by the experiences of the students, to situations where they are presented with obstacles and contradictions, which places them in a conflict and breaks with the expectations that were expected, thus trying to get them to reflect on the situation, in order to achieve the objectives set.

The interventions that were made and the use of manipulative material that was used when carrying out the activities in the classroom, resulted in the pupils carrying out the final evaluation activity on their own, without major problems, making sense of the graphic, verbal and numerical representation, despite the fact that these were the first activities in which they used fractions.

As for the fractions in the sequence, a change of reference unit from a whole to a half as a reference unit is handled intuitively, this is achieved by dividing the circle into halves, making reference to the fact that a quarter represents half of a half and an eighth half of a quarter.

It is considered that giving an initial meaning to half of a half and half of a quarter can contribute later on to give meaning to the multiplication of fractions and thus avoid a recurrent confusion in students when performing this operation, as they will be able to associate it to the action they are carrying out in this activity, by giving the multiplication sign (x) the meaning of, i.e. 1/2 x 1/4 means 1/2 of 1/4, in this way, they can explain why when multiplying two fractions the product is reduced, contrary to what happens when multiplying two natural numbers.

In addition, the design based on properly articulated problem situations and the role of the teacher as a guide to the activity, led the students to use their previous knowledge to provide answers to the problems posed.

With regard to the construction of knowledge about the use of halves, quarters and eighths to express orally and in writing different measurements and divisions, the activities were a great support; firstly because the pupils were discovering these new numbers through the questions, making sense, firstly of what was being divided through the actions of: folding, colouring, cutting out and superimposing, then how they were expressed verbally, and finally numerically. This became visible when carrying out the assessment in the last part of the sequence; in this, in spite of using figurative representations different from those used throughout the sequence and in some cases changing the units of reference to larger than the unit unit, the students were able to give answers to what was asked of them, with minimal doubts.

Conclusions

Throughout this work, fractions were studied, taking into account different perspectives and analyzing the ideas of various authors. From these studies it was concluded that it is necessary to create material to support the learning of fractions.

The didactic proposal presented in this work helps students to make sense of fractions at an early stage, that is, in their introduction, using manipulative material, since, as has been pointed out, the incorporation of manipulative material to support the learning of fractions is a good tool, given that:

- It catches the students' attention.
- By manipulating the material they can associate the part with the whole.
- Manipulation allows them to validate their answers.
- It allows them to confirm their ideas.
- They are able to schematize the parts of fractions through figurative representations.
- They are able to make comparisons between fractions through their different graphical, verbal and symbolic representations.

- They can interact with their classmates clarifying or defending their procedures and answers making use of manipulative material as a reference for their arguments.

- They are able to do a greater amount of autonomous work.

A recurrent problem, not only in the learning of fractions, but of any content, is the lack of interest in reading and, consequently, the students' lack of reading comprehension. This problem could be mediated with the design of activities that included short questions and images.

The question remains about the type of material that students have to develop reading competence; although this is a problem that does not fall squarely within the interest of educational mathematics, it also warns about the quality of official educational materials used by students and teachers to support this aspect.

In concluding this work, it is confirmed how important it was to choose the first moment in which fractions are studied as the focus of the design, because, without neglecting the importance of the other moments, we had the satisfaction of having lived the experience of accompanying the students in the first construction of meanings.

With regard to teachers, their work requires more planning, which is sometimes difficult for them due to lack of time, interest, knowledge, among others. It would therefore be interesting to be able to design guided activities so that teachers can get an idea of what they can do in the classroom.

These guides, made by a team of experts as a long-range project, could focus on each of the meanings of the fraction, covering each of the expected learning at the basic, intermediate and higher levels.

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