

Didactic proposal using technology for learning of Special Products in High School

Propuesta didáctica con uso de tecnología para el aprendizaje de productos notables en Bachillerato

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Abstract

Learning Special Products represents for high school students a tool that facilitates problem solving throughout their education, which makes it a fundamental topic in elementary education. The general aim for this study was to determine the impact of a didactic proposal using technology to improve the understanding of Special Product in first semester high school students, to improve the failure rate. A total of 30 students participated in the study, of which 18 were from the control group and 12 students from the experimental group. To collect information, three instruments were used: a diagnostic evaluation, the didactic sequence, and a final evaluation. The design for this research was quantitative and quasi-experimental. The results showed that the didactic proposal helped in the understanding of the Special Products, thus achieving that the students could carry out the conversion of the graphic representation to the algebraic and from the algebraic to the verbal. In addition, an improvement was obtained with respect to decreasing the failure rate of the mathematical object.

Technology, Special Products, High school

Resumen

El aprendizaje de productos notables representa para los estudiantes de bachillerato una herramienta que facilita la resolución de problemas a lo largo de su educación, lo que lo hace un tema fundamental en la educación básica. El objetivo general de este estudio fue determinar el impacto de una secuencia didáctica con el uso de la tecnología para mejorar la comprensión de productos notables en los alumnos de primer semestre de bachillerato, con la finalidad de mejorar el índice de reprobación. En el estudio participaron un total de 30 estudiantes, de los cuales 18 fueron del grupo control y 12 estudiantes del grupo experimental. Para recolectar la información se utilizan los instrumentos de evaluación diagnóstica, secuencia didáctica y la evaluación final. El diseño para esta investigación fue del tipo cuantitativo y cuasiexperimental. Los resultados mostraron que la propuesta didáctica ayudó en la comprensión de los productos notables, logrando con ello que los alumnos pudieran llevar a cabo la conversión de la representación gráfica a la algebraica y de la algebraica a la verbal. Además, se obtuvo una mejora con respecto al índice de reprobación del objeto matemático.

Tecnología, Productos notables, Bachillerato

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Introduction

The teaching of mathematics represents one of the areas of study with the highest expectations within the world of education. Society increasingly demands an effective mathematical and scientific culture that guarantees a favorable behavior in citizens, despite the contradictions that exist among politicians who claim to have no taste or mastery of mathematics (Artigue, 2004). Algebra is the means for students to acquire more complex mathematical knowledge, and as Esquinas (2009) expresses it, its study generates difficulties in students due to the change in the use of language and methodologies and procedures that they tend to confuse.

Currently, upper secondary education requires and demands learning methodologies that guarantee the conception of certain mathematical concepts that potentiate the ability to develop and solve problems of a high degree of complexity; therefore, it is important to strengthen basic topics such as remarkable products, a topic that begins its study in secondary education (SEP, 2017).

It is very common to see arithmetic deficiencies in students, most of them do not master the multiplication tables and consequently cannot divide; therefore, they cannot develop algebraic algorithms, this has an impact on learning in the remarkable products, since the knowledge goes in sequence and is indispensable for the understanding of algebra. In addition, among the situations that show more difficulties for the student to understand the remarkable products are, the little knowledge they have regarding the law of signs, confusion in the reduction of like terms, the lack of ability to identify the different remarkable products and the inadequate application of products and quotients, as well as factoring (Caiminagua and Villa, 2016).

Carrasco, Quichua and Tullume (2018), point out deficiencies in the learning of remarkable products and factorization, this problem arises from the use of some inadequate strategies such as the direct application of unattractive remarkable formulas by teachers and for disregarding the importance of identifying about the students' previous knowledge when they are introduced to these topics in their first year of high school.

The topic of remarkable products is often difficult to understand and is a barrier in the study of algebra topics; other difficulties in the teaching and learning of remarkable products highlight the way in which teachers teach this topic, which is usually taught in a decontextualized way, limiting a true practical application (Sánchez, 2011).

The learning of remarkable products represents for high school students a tool that facilitates problem solving throughout their education, which makes it a fundamental topic in basic education. Carrasco, Quichua, and Tullume (2018), comment that the methodology for teaching remarkable products by showing the correct steps in solving exercises, e.g., directly solving

$$(a + b)^2 = a^2 + 2ab + b^2$$

without demonstrating that this product comes from a multiplication of

$$(a + b)(a + b)$$

Has led to students' disinterest in remarkable products. When developing an exercise directly from the formula they relate it to the memorization of a practical system that only some students can master.

Solórzano and Fuentes (2016), point out that based on experiences of teaching practices, they have identified the problems that exist in the transition from arithmetic to algebra, specifically when it comes to manipulating variables, therefore they propose a didactic strategy that favors a change in algebra topics and specifically from the binomial to the square, as well as topics that derive from the distributive property and the perfect square trinomial, which can counteract the problems that exist not only in the solution but also in the interpretation of the solution by means of manipulative didactic material. In research on the difficulties in solving remarkable products, Méndez (2008) states that students have their own methodology to solve remarkable products and implement spontaneous and persistent action models when solving this type of exercises, identifying some cases of erroneous resolution for the squared binomial developing it as a difference of squares, for which he proposes a formal contextualization that allows the correct identification and development of the product of two binomials.

The use of technology as a didactic tool in the teaching and learning of the remarkable products could make a difference in achievement, despite the fact that students have their own equipment to work in the classroom, the introduction of technology in the classroom has modified the teaching of mathematics in all senses, unlike the algorithmic approach that this discipline usually has, technology encourages the character of reflection and discovery (Araya, 2007).

Technology represents a very important place in mathematics because it generates in students analysis and exploration skills, as well as problem solving applied in extra-mathematical contexts. In addition, it allows more realistic scenarios or situations facilitating visual comprehension, allowing students to access complex content that would otherwise be very difficult to explore, in other words, technology comprises tools that can be used for the design and application of illustrative activities allowing meaningful learning (Padilla-Escorcia and Conde-Carmona, 2020).

The benefits of implementing the use of technology in education are the motivation or the challenge felt by teachers to create more attractive work environments for their students, thus promoting educational innovation, and it also favors collaborative learning among students and promotes self-study by having all kinds of information at hand (Levicoy, 2014).

For his part, Ayala (2020) mentions that software development is a tool that helps students to increase learning in topics such as remarkable products and factorial decomposition, and also comments that the interactive elements used, such as messages implemented in the software are appropriate to provide feedback to students. Adriano (2019) comments that, due to the advances that technology has had, it has been established as a fundamental tool for the appropriation of skills in students, since thanks to it the teacher has within reach resources where notable products and factorization calculations can be exemplified facilitating their learning.

The techniques used by teachers limit students to the memorization of remarkable formulas and mechanisms that do not guarantee significant learning of this subject of study.

Remarkable products are used in more advanced courses, so it is necessary to develop didactic strategies that improve and guarantee their understanding. In this sense, the present study proposes the design of a didactic sequence with the use of technology that allows the learning of the contextualized development of the squared binomial and the product of conjugate binomials. Based on this, the following research question is posed: What is the impact of a didactic sequence with the support of technology for the understanding of the squared binomial and conjugate binomials in high school students?

Objectives

General Objective

To determine the impact of a didactic sequence with the use of technology to improve the understanding of remarkable products in first semester high school students, in order to improve the failure rates of this subject of study. Specific objectives

- a. To identify the impact of a didactic sequence with the support of technology for the understanding of the squared binomial and difference of squares in high school students.
- b. The implementation of a didactic sequence will help to improve the failure rate of the mathematical object.

Hypothesis

The implementation of a didactic sequence of squared binomial and conjugate binomials based on Raymond Duval's theory of semiotic representations will help students to improve their understanding of the mathematical object.

Justification

Commonly in the process of teaching mathematics, and specifically the remarkable products, the student is taught to memorize the rules and formulas, so that students can solve in the short term simply by observation and identification of each one.

This methodology is successful with a reduced number of students, and in particular reduces the possibility for the student to develop critical thinking, by not succeeding with the traditional way of solving remarkable products in a memoristic context, the student looks for other alternatives for their solution, usually with basic algebraic operations, which extends the activity causing frustration and demotivation in the student (Chilan, 2019).

Remarkable products are a fundamental part in the study of mathematics, they are not only used to solve basic problems in the first levels of algebra, but also have application in the solution of second degree equations, in conic sections among other topics, so it is important to take care in the development of their understanding (López, 2014). Students must adequately understand the remarkable products, which as mentioned by Vega-Castro (2010), the importance of mastering the remarkable products lies in continuing with success in various topics of algebra contained in the curriculum of educational institutions at the high school level.

If a student knows how to repeat a demonstration, but does not know how to repeat it if the letters or the position of the polygon are changed, it means that he has learned the demonstration by heart and this has no value. Rather, it has a highly negative value, since it means that the student not only ignores the demonstration, but that he is totally unaware of what mathematics is and that he has wasted the use of memory for a useless and uneducational purpose (Santaló, 1962 cited by Crespo, 2005, p. 27).

The type of tasks that the remarkable products allow to solve are many, among which stand out: Calculating products of algebraic expressions, simplifications of algebraic expressions, factoring, development of algebraic expressions, finding values of different variables, finding the intersections with the axes, roots of a function, distance between two points, finding the general form of a conic, determining the position of the curve of conics, determining both vertical and horizontal asymptotes and integration by the method of partial fractions.

In fact, the remarkable products are the connection to factoring, which is an essential part of algebra and is a necessary support knowledge for learning and teaching different topics in mathematics (Martos, 2013).

Among other benefits that the student acquires in the learning of remarkable products is the strengthening of mental agility, logical reasoning allowing autonomy and development of cognitive skills (Alcívar and González, 2018).

Currently, the number of first semester high school students with problems in developing remarkable products is very high. This, motivates the interest in the design and implementation of a didactic proposal supported with technology that allows the student to improve the performance in class regarding the remarkable products.

Theoretical Framework

Semiotic representations play a fundamental role in mathematics; on the one hand, they favor understanding and on the other hand, they motivate mediation with mathematical objects. "Semiotic representations are productions constituted by the use of signs that belong to a system of representation which has its own definitions of meaning and knowledge" Duval (1999). Representation is frequently used in its verbal form (representing), however, there are different types of representation, among them are the mental ones, which are associated with the set of images and the conceptions that an individual may have about an object or a situation, on the contrary, semiotic representations are productions constituted by the use of signs that belong to a system of representation which has its own meaning.

Duval (1993) mentions that in the learning of mathematics it is necessary to distinguish between a mathematical object and its representation, in such a way that the understanding of the object is considered conceptual, since they cannot interact directly, so it is necessary to use semiotic representations to be able to perform some kind of activity on mathematical objects, calling semiosis to the apprehension or production of a semiotic representation and noesis to the conceptual apprehension of an object, pointing out that they are two inseparable activities.

Furthermore, he characterizes a semiotic system as a representation system which can be an RRS if it allows the three essential cognitive activities linked to semiosis:

Formation: where the representations of a particular register allow expressing an object as the presentation of something in a system and refers to the signs that characterize each register and its identification rules in the same register, i.e. a formula can be identified in the algebraic register or in the statement in its verbal register, for example, if one wishes to express the binomial squared in the algebraic register it can be expressed as $(a + b)^2$.

Processing: when a representation is transformed into another representation, but within the same representation register, for example

$$(a + b)^2 = a^2 + 2ab + b^2$$

This equality describes the cognitive activity of processing.

Conversion: where a representation register is transformed into a different register, but part of the initial meaning is preserved, in this activity there is no rule, the conversion is an external transformation to the initial register, which can be carried out when the student faces a problem situation that is expressed verbally and the transition is made to a symbolic writing by making the statement of an equation (Table 1).

From the point of view of Duval (2016), in order for understanding to occur in the learning of mathematics, the object must be distinguished from its representation, referring with this to not confusing mathematical objects, with their representations, i.e., numbers, functions, straight lines, symbols or their graphics since the mathematical object can occur through very different representations.

In the process of teaching mathematics and the beginnings of algebra, the significant change that exists with respect to the study of arithmetic, the substitution of numbers for letters and the change that occurs in the language with the students, generates confusion and difficulty in the concepts that are introduced.

Remarkable products represent an important part of so many applications of algebra, especially the solution of problems involving remarkable products, which are defined as "certain products that meet fixed rules whose result can be written by simple inspection, i.e. without verifying the multiplication" (Baldor, 2011, p. 97).

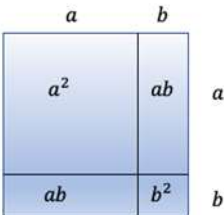
Registration	Representation
Verbal	The square of the sum of two terms will always be equal to the square of the first term, plus the double product of the first term by the second term plus the square of the second term.
Algebraico	$(a + b)^2 = a^2 + 2ab + b^2$
Graphic	

Table 1 Different registers of representation of the squared binomial

Mathematics education registers studies on the difficulties of learning mathematical objects that are not found in other areas of education. This unique form that the study of mathematics has leads to the analysis of the cognitive processes that revolve around its understanding, the origin of the differences that separate mathematics from other areas of education is precisely the way in which the objects studied are accessed (Duval, 2012). Within the execution and study of the remarkable products, confusion is generated in the development of the squared binomial algorithm with the development of the product of conjugate binomials, where the student forms a difference of squares as the product of two binomials. In this research, specific emphasis is placed on the development of these two products through a didactic proposal designed based on the theory of RRS.

According to Liberos (2017), due to the complexity of the study of remarkable products, with the consideration of the knowledge and the way of explaining of the teacher, who sometimes follows a pattern in terms of the solution method, and in very few occasions it is demonstrated with another type of representation different from the algebraic.

Which in the future would facilitate the understanding and solution of problems involving the application of remarkable products. The same author comments that it is necessary to find a way to implement activities related to resources that promote the different types of representation, which is why it is a topic that provokes interest in the development of a didactic sequence that can improve learning by promoting cognitive activities, particularly the processing and conversion of at least two representation registers according to the theory of RRS.

For Díaz-Barriga (2013), a didactic sequence is an organization of learning activities that are carried out with students with the purpose of creating situations that allow them to develop meaningful learning. In addition, it is a tool that, in order to be created and used, the teacher must have a mastery of the subject, understanding of the syllabus, demand to have a degree of pedagogical experience and vision, as well as their ability to devise activities for student learning.

Tobón, Prieto and Fraile (2010) describe a didactic sequence as an organized set of learning activities that, with the guidance of the teacher, can achieve academic objectives by considering significant resources during practice. This reflects strong results in the training of students since the work is done on learning goals and allows measuring and reinforcing knowledge.

As established by Díaz-Barriga (2013), a didactic sequence should consist of 3 activities: (a) the opening activities which have the purpose of identifying previous knowledge and using it as a basis for new knowledge, likewise it is pertinent that the teacher clarifies any doubts that arise at this stage to be able to start with new knowledge; (b) the development activities, in which students begin to interact with new knowledge or with the theoretical part; (c) the closing activities, in which previous knowledge is integrated with new knowledge, the learning processes developed during the sequence. The didactic sequence is a process in which the teacher must have the ability to make last minute changes that modify the sequence depending on the progress made by identifying the difficulties that students may present.

Linked to the importance of the development of a didactic sequence that can improve learning by promoting cognitive activities, it is relevant to mention that technology can motivate students to study mathematical concepts, since there are different free-use software that facilitate the transition between the different representation registers. The application of different representations for the solution of a mathematical problem favors the understanding and helps to solve it, when a student makes use of the different representations to solve a problem, he is making use of all the heuristic mechanisms (Libreros, 2017).

Information and communication technologies (ICT's) represent a set of techniques, developments and advanced devices, everything that represents applied technology such as software and hardware used to create, store, exchange or process information in different forms, such as audios, photographs, websites, among others (Tello, 2007). The implementation of ICT's in education is an increasingly important resource as it has led to transform the teaching methodology, so the teacher must be capable both in his pedagogy and in the use of technology for education (Luna, 2017).

The use of technology in education has lately revolutionized the way in which classes are taught, so that the use of these techniques in the classroom goes from being a possibility to be instituted as a necessity and as an indispensable didactic work tool for the teacher (Gómez and Macedo, 2010). Of the most used didactic software in the school environment, specifically in mathematics has been GeoGebra, where students can evidence the way in which their mental process is externalized and thereby make them more understandable by performing all kinds of activities related to dynamic geometry and algebra, managing to relate the RRS (Espinoza, 2019). According to Iranzo and Fortuny (2009), the use of GeoGebra promotes geometric thinking in students that facilitates the visual, algebraic and conceptual support required when studying a mathematical object, which allows solving problems in another way.

Research Methodology

Participants

The present study was conducted in a high school in southern Sonora that had 336 students enrolled, of which 112 were first semester students. The participants in the study were students from groups A and B, which consisted of 18 and 12 students, respectively. Group A consisted of 17 male students and one female student, while group B consisted of nine females and three males, whose ages ranged from 15 to 16 years old.

Instrument

The didactic sequence consisted of four activities and involved three representation registers, verbal, algebraic and graphic. The didactic sequence contained instructions for manipulating GeoGebra Applets, which in turn allowed the graphical construction of the product of two squared binomials and conjugate binomials.

It also included concepts and definitions about the mathematical object, as stipulated by Duval (1993), learning mathematics involves the execution of cognitive activities such as conceptualization, reasoning and problem solving, and the representation of different registers is necessary.

Design

This research is of the quantitative type, since through an organized process it allows the demonstration of certain hypotheses raised at the beginning of this research. In addition, it is of the experimental type since the variables under study are manipulated; in this case, the learning of the squared binomial and squared difference were manipulated to measure their effect on the student's performance, measured through the grade on the exam.

Experimental research is also classified into pre-experiments, "pure" experiments, and quasi-experimental (Hernández-Sampieri & Mendoza, 2018). In this research the subjects are not randomly assigned, they are groups that are already formed before the experimentation, which authenticates the design as quasi-experimental.

In addition, the study is transectional because it is carried out in a single period. The control in an experiment achieves internal validity and this can be obtained from the comparison of two or more groups. When there are two comparison groups, one group is the experimental group (EG) and the other is the control group (CG). In these studies, pre (diagnostic) and post (final) tests are used, which allow analyzing the evolution or progress of the groups (Hernández-Sampieri and Mendoza, 2018). In this research the control group was formed by group A and the experimental group by group B.

Procedure

The procedure followed for the development of this research is described below.

1. Development of the diagnostic evaluation. A test was designed to measure the previous knowledge that the students had before the implementation of the didactic sequence, being a mathematical object that begins its study from basic education (high school) it is likely that students had some notions about the subject and it was important to know the level at which they were prior to the study.
2. Elaboration of the final evaluation. The evaluation corresponding to the partial evaluation is carried out, which includes exercises of the same type that are evaluated in the exam prior to the implementation of the didactic sequence.
3. Application of the diagnostic exam. This stage was carried out at the beginning with the application of the diagnostic exam of groups A and B, with a duration of 20 minutes for the solution, through a technological platform used in the institution and during a synchronous session through video link in Zoom due to the sanitary contingency.

4. Implementation of the didactic sequence. The sequence is implemented with a duration of 90 minutes remotely through the Zoom platform, students performed a total of 4 activities guided by the teacher, corresponding to the squared binomial and conjugate binomials from the graphical representation of the object to subsequently perform the treatment and finally reach an algebraic and verbal representation of the same mathematical object.
5. Application of the final exam. The final exam was applied where 4 exercises corresponding to the mathematical object were included, also interacting through a videoconference.
6. Analysis of results. The results of the diagnostic evaluation, as well as the final evaluation were analyzed by means of measures of central tendency and dispersion, frequency tables and statistical hypothesis tests for the comparison of the groups, specifically, the Student's t-test for independent samples and related samples was used.

Results

A test of equality of averages for the diagnostic test of the control group (A) and experimental group (B) was performed using Student's t-test (Table 2) for independent samples. As can be seen, the test was not significant, so the averages were statistically equal for both groups, showing that the groups show similar academic performance. The average grade of the control group was 8.05, slightly higher than the average grade of the experimental group, which was 7.58.

t	gl	Sig. (bilateral)	Difference in averages
.514	28	.611	.47222

Table 2 Student's t-hypothesis test for independent samples to test equality of averages in the average grades between the control group and the experimental group for the diagnostic exam

The Student's t-test (Table 3) for independent samples was used to test the equality of averages for the final exam for the experimental group and the control group.

The test was significant, so the averages were statistically different for both groups. The average grades of the experimental and control groups in the final exam were 8.98 and 7.04 respectively, almost two points more between both grade averages.

This shows that the experimental group performed better academically than the control group and, therefore, proves the effectiveness of the didactic sequence. With respect to the standard deviations, those of the control group were larger (2.53) than those of the experimental group (0.89).

t	gl	Sig. (bilateral)	Difference in averages
-2.532	28	.017	-3.50725

Table 3 Student's t hypothesis test for independent samples to test equality of averages in the average grades between the experimental group and the control group in the final exam.

Finally, the distribution of percentages of students who achieved a grade of 7 or higher (sufficient achievement) in the diagnostic and final evaluations of the control and experimental groups, as well as those who had a grade lower than 7 (insufficient achievement) were obtained (Table 4). The failure rate of the mathematical object under study improved considerably, since in the diagnostic exam the experimental group had 25% of students with insufficient achievement and in the final exam there were 0% of students with insufficient achievement.

		Insufficient utilization	Sufficient utilization
Diagnostic test	Control group	11	89
	Experimental group	25	75
Final exam	Control group	67	33
	Experimental group	0	100

Table 4 Distribution of percentages of students who had insufficient and sufficient achievement, of the control and experimental groups, of the diagnostic and final exams

Conclusions

After the implementation of the didactic sequence and as a result of this study, it was possible to observe a significant difference in the grades of the final exam between the experimental and control groups, considering that in the diagnostic exam the average of both groups was not significant.

This shows the positive effect that the didactic sequence had on the students' learning of the binomial squared and the difference of squares, coinciding with what Tobón, Prieto and Fraile (2010) point out, a didactic sequence can achieve the academic objectives considering significant resources during the practice, which reflects strong results in the students' formation.

The study shows that this didactic proposal achieved that students could carry out the conversion from graphic to algebraic representation and from algebraic to verbal, which benefited the understanding of the remarkable products under study, since as mentioned by Duval (1993) it is important that students can convert representations from one system to another quickly and spontaneously, which he defines as a coordination between representation registers being the necessary activity for the conceptual apprehension of mathematical objects.

The use of communication tools represented a great support during the implementation of this didactic proposal, the application it has in education has lately revolutionized the way in which classes are taught, mainly due to the health contingency our country is going through, so that as mentioned by Gómez and Macedo (2010), the use of these techniques in the classroom goes from being a possibility to become a necessity and an indispensable didactic work tool for the teacher.

The use of Geogebra facilitated the concrete and dynamic vision of the graphical representations of the remarkable products under study, improved the differentiation between the squared binomial and the conjugate binomial, and the student was able to recognize the mathematical object in all representation registers.

With respect to the failure rate, in the diagnostic exam, the control group obtained a lower percentage of students with insufficient achievement, while in the final exam, the experimental group had no students with insufficient achievement. This again shows the effectiveness of the didactic sequence.

Recommendations

As recommendations, it can be suggested to expand the application of context problems, where the student can achieve the conversion of the algebraic register to the graphic and vice versa, since it was in this type of exercises where the greatest confusion and error was noted.

It is suggested the implementation of this didactic proposal in a larger sample, which allows to have a smaller margin of error.

It is recommended that the didactic sequence be implemented in other educational institutions, both in high school and elementary school, since it is a mathematical object that begins its study in secondary education, and a comparison can be made to identify if it requires any improvement.

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